Novel Solution Approach for Optimizing Crude Oil Operations

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Scheduling of crude oil operations is a complex nonlinear problem, especially when tanks hold crude mixes. We present a new mixed-integer nonlinear programming (MINLP) formulation and a novel, mixed-integer linear programming (MILP)–based solution approach for optimizing crude oil unloading, storage, and processing operations in a multi-CDU (crude distillation unit) refinery receiving crude from multiparcel VLCCs (very large crude carriers) through a high-volume, single-buoy mooring (SBM) pipeline and/or single-parcel tankers through multiple jetties. Mimicking a continuous-time formulation, our primarily discrete-time model allows multiple sequential crude transfers to occur within a time slot. It incorporates several real-life operational features including brine settling and tank-to-tank transfers, and is superior to other reported models. Notably our algorithm avoids concentration discrepancy and MINLP solutions by identifying a part of the horizon, for which its linear relaxation is exact, and then solving this MILP repeatedly with progressively shorter horizons. By using 8 h time slots and a hybrid time representation, an attractive approach to this difficult problem is presented.
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Introduction

Petroleum refining involves separating crude oil into its constituents, and converting and treating them into marketable products. In recent years, refining has become an extremely competitive business, characterized by fluctuating demands for products, ever-changing raw material prices, and the incessant push toward cleaner fuels. To survive financially, a refinery must operate efficiently. From an operational perspective, a plant would operate the best in a steady state with consistent feedstocks and product requirement and all units operating at full capacity. Any change is undesirable because it may lead to off-spec products, reduced throughputs, increased equipment wear and tear, uncertainty, and more work. Nevertheless, in the current competitive environment, profit depends on agility, that is, the ability to exploit short-term opportunities to fill demand at higher profit margins. Processed crude compositions have the greatest influence on refinery margins. Therefore, refiners tightly control the quality of crude charge and use advanced technologies to plan and schedule crude oil changes.

Optimization plays an important role in managing the oil refinery. Oil refineries have used optimization techniques for a long time, especially successive linear programming (LP) for the planning and scheduling of process operations. Although the planning systems provide coordination over several months, the scheduling systems plan the activities over days to weeks. Planning precedes scheduling and uses forecasted product demands. Crudes are purchased based on the monthly refinery plan. Scheduling subsequently accounts for deviations from the forecast and accounts for changes in demand or plant capability. Many refineries partition scheduling into crude scheduling, hydraulic scheduling, and product scheduling (Bodington, 1995). Crude schedulers react to the crude arrivals, assign destination tank/s for each crude, blend crudes as needed to meet the targets for yields and qualities of the fractionated products from the crude distillation unit (CDU), and determine

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the charging rate to each crude unit. Hydraulic scheduling involves the operations of major units, and inventories between the units with a view to properly control intermediate inventories. Product scheduling is concerned with the blending and distribution of final products, while ensuring the inventory control. Detailed modeling, effective integration, and efficient solution of these three scheduling problems is essential for the scheduling of overall refinery operations. In this article, we address the first problem of crude scheduling and consider the rest in our future work. A priori information about the procured crudes, including their types, quantities, and expected arrival times at the refinery, for example, is used to schedule short-term activities such as unloading crude oil from vessels to storage tanks and charging various mixes of crude oils to each distillation unit subject to capacity, flow, properties, and composition limitations.

The schedulers' job in a refinery has become increasingly complex in recent years. They must continuously watch both the crude oil movements and the operational status of the plant and match them to fluctuating demands. In most cases, under intense time pressure and low inventory flexibility, the schedulers rely largely on their experience, and select the first feasible solution found by a spreadsheet model or other tool. However, tremendous opportunity for economic and operability improvement exists in this process. Quantifiable economic benefits from a better scheduling are improved options, increased utilization and throughput, intelligent use of less-expensive crude stocks, capture of quality barrels, reduction of yield and quality giveaways, improved control and predictability of downstream production, reduced demurrage costs, and so forth. Kelley and Mann (2003a,b) describe crude scheduling as an application with multimillion dollar benefits and quantify these benefits.

Pelham and Pharris (1996) pointed out that, although planning technology can be considered as well developed, fairly standard, and widely understood, the same could not be said for short-term scheduling. There is a need to improve scheduling models to account for the complexity arising from discrete decisions and the various blending relationships. This is a complex problem requiring simultaneous solutions to crude flows, allocations of vessels to tanks, tanks to CDUs, and calculation of crude compositions. Kelley and Mann (2003a,b) discussed the intractability of this problem in general, especially in a reasonable time.

Shah (1996) reported a discrete-time mixed-integer linear programming (MILP) model for crude oil scheduling by separating it into two subproblems. The upstream problem included portside tanks and offloading and the downstream problem included allocation of charging tanks and CDU operation. The objective was to minimize the tank heel. Almost concurrently, Lee et al. (1996) also reported a MILP model to minimize operating cost arising in crude oil unloading, tank inventory management, and crude charging. They used single-crude storage tanks and mixed-crude charging tanks in their configuration, but did not allow splitting of feed to multiple CDUs or multiple tanks charging one CDU. They ensured feed quality by using constraints on the concentration of one key component in charging tanks, but did not consider some real-life operational features such as brine settling, multiple-parcel vessels, and multiple jetties, for example. Furthermore, they processed prefixed ranges of crude mixes in charging tanks one after another to meet the total demand. However, as we show later by some motivating examples, the most important limitation of their work is the composition discrepancy in their schedules, which arises from their linearization of the bilinear terms resulting from the charging of crude blends. Recently, Li et al. (2002), recognizing this composition discrepancy, proposed an iterative MILP–NLP (nonlinear programming) combination algorithm to solve the problem. They also attempted to reduce the number of binary decisions by disaggregating the tri-indexed binary variables into bi-indexed ones, and incorporated new features such as multiple jetties and two tanks feeding a CDU. However, their algorithm requires solving a NLP at each iteration, and as we show later, may fail to obtain a solution, even when a solution exists. Furthermore, their changeover definition leads to double counting and their allocation variables impose undesirable restrictions on charging and unloading options.

Kelley and Mann (2003a,b) suggested decomposition as a possible strategy for solving this complex problem, in which allocation logic and quality accounting are done in two separate steps. However, they correctly anticipated that such a strategy might fail to yield a solution in some cases. This article avoids this pitfall by using a single-step, iterative MILP approach. As seen from the above, none of the existing approaches satisfactorily addresses composition discrepancy in crude charge to CDU, transfer lines with nonnegligible volumes, and other important features such as settling time. In this article, we propose a novel, hybrid (discrete-continuous) formulation that incorporates these industrially important configurational and operational features. We also use a realistic profit-based objective function that includes crude margins, safety-stock penalties, and accurate demurrage accounting. Finally, we devise an iterative, MILP-based solution algorithm that obviates the need for a NLP solution.

After defining the problem in the next section, we use several motivating examples to highlight the drawbacks of existing approaches and the need for further work. For the sake of simplicity and to improve readability, we begin with a mathematical programming formulation for scheduling a refinery with one SBM line, I storage tanks, and U crude distillation units (CDUs). Next, we discuss the modifications required to handle J jetties instead of one SBM line. Subsequently, we merge the two to obtain a formulation that accommodates one SBM line and J jetties. Then, we extend the model to allow a common practice of tank-to-tank transfers. Finally, we present our solution algorithm, compare and contrast our model with previous models, and demonstrate application of our approach using several diverse examples.

Problem Statement

Most refineries receive and process several crudes. Marine transportation is common for crude oil. Although pipeline-access refineries do exist, we focus in this article on a typical coastal, marine-access refinery only because of our geographical location and our interaction with such a refinery. Figure 1 shows the crude oil unloading, storing, and processing in a typical marine-access refinery. The configuration involves crude offloading facilities such as an SBM (single-buoy mooring) or SPM (single-point mooring) station and/or one or more jetties, storage facilities such as storage tanks, and/or charging

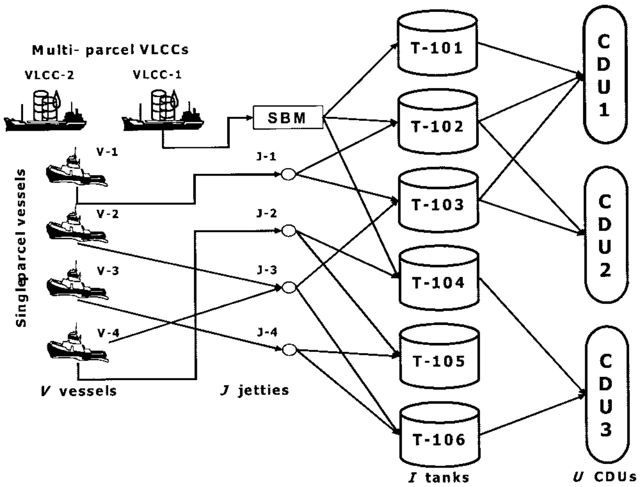

Figure 1. Schematic of oil unloading and processing.

tanks and processing facilities such as CDUs. The operation involves unloading crudes into multiple storage tanks from the ships/tankers arriving at various times and feeding the CDUs from these tanks at various rates over time. Thus, the problem involves both scheduling as well as allocation issues.

Crudes arrive in either large multiparcel tankers or small single-parcel vessels. A very large crude carrier (VLCC) has multiple compartments to carry several large parcels of different crudes. However, because of its huge size, a VLCC must dock offshore at a station called SBM or SPM, which connects to the crude tanks in the refinery by one SBM pipeline. SBMs have become quite important because transporting large crude parcels reduces unit transportation costs. Even for a coastal refinery, the SBM line has a significant holdup that cannot be ignored, given that the crude type present in the line may not match the crude type of the parcel currently being unloaded from a multiparcel VLCC. In practice, pipelines also transport crude from marine terminals to distant inland refineries or various petroleum products from refineries to faraway destinations (Rejowski and Pinto, 2003). Although we are not addressing the latter situation in this article, our proposed approach for handling pipeline transport can address the former.

However, from time to time, a refinery may also receive small parcels of single crudes by small ships that dock at an onshore jetty. A refinery may have multiple such jetties. The characteristics and operations of SBM and jetties are quite different. Usually, there is only one SBM, so VLCCs can dock only one at a time. Similarly, there is only one pipeline, so only one crude parcel can unload at any time. In addition, each parcel must first eject the crude already present in the SBM line. In contrast to the SBM line, the holdup in the pipeline connecting a jetty to a tank is negligible. When there are multiple jetties, multiple ships can dock at the same time and simultaneously transfer crude parcels.

Many types of crude exist in the market, varying widely in properties, processability, and product yields. Years of experience have helped the refiners classify crudes based on some key characteristics such as processability, yields of some premium products, impurities, or concentrations of some key components that influence the downstream processing. This has led to the common practice of segregating crudes (Kelly and Forbes, 1998) in both storage and processing. Thus, tanks and CDUs usually store or process only specific classes of crudes.

With the above brief introduction, we now state the crude scheduling problem as addressed in this article.

Given:

(1) Arrival times of ships/VLCCs, volumes, and crude types of their parcels

(2) Configuration details (numbers of CDUs, storage tanks, jetties, and their interconnections) of the refinery

(3) Holdup in the SBM pipeline and initial crude type

(4) Limits on flow rates from the SBM station and jetties to tanks and from tanks to CDUs

(5) Limits on CDU processing rates

(6) Storage tank capacities, their initial inventory levels, and initial volume fractions of crudes in tanks

(7) Information about modes of crude segregation in storage and processing

(8) Information about key component concentration limits during storage and processing

(9) Economic data such as sea waiting costs, pumping costs, crude changeover costs, and so forth

(10) Production demands during the scheduling horizon. These are normally available from the monthly production plan of the refinery

Determine:

(1) A detailed unloading schedule for each VLCC/vessel

(2) Inventory and composition profiles of storage tanks

(3) Detailed crude charge profiles for CDUs

Most refiners use some operating rules. In this work, we assume the following:

(1) A tank receiving crude form another tank, a ship, or a tanker cannot feed a CDU at the same time.

(2) Each tank needs some time (8 h) for brine settling and removal after receiving crude.

(3) Multiple tanks can feed a single CDU. Most refiners allow at most two tanks to feed a CDU because the operating complexity increases and controllability becomes a problem for more than two tanks.

(4) A tank may feed multiple CDUs. Again, a tank normally does not feed more than two CDUs.

Finally, we make the following assumptions regarding the refinery operation:

(1) Only one VLCC can unload at any moment. This is reasonable, given that there is only one SBM.

(2) The sequence in which a VLCC unloads its parcels is known a priori. A VLCC always unloads parcels in the same sequence in which it loads (Singapore Refining Company, 2003). Refinery planning, which usually takes place weeks before the ship starts sailing toward the refinery and the scheduling activity, predefines the loading sequence of parcels. Thus, the parcel unloading sequence is indeed beyond the purview of crude oil scheduling and this assumption is reasonable.

(3) A parcel can unload to only one storage tank at any moment.

(4) The SBM pipeline holds only one type of crude at any time and crude flow is plug flow. This is valid because parcel

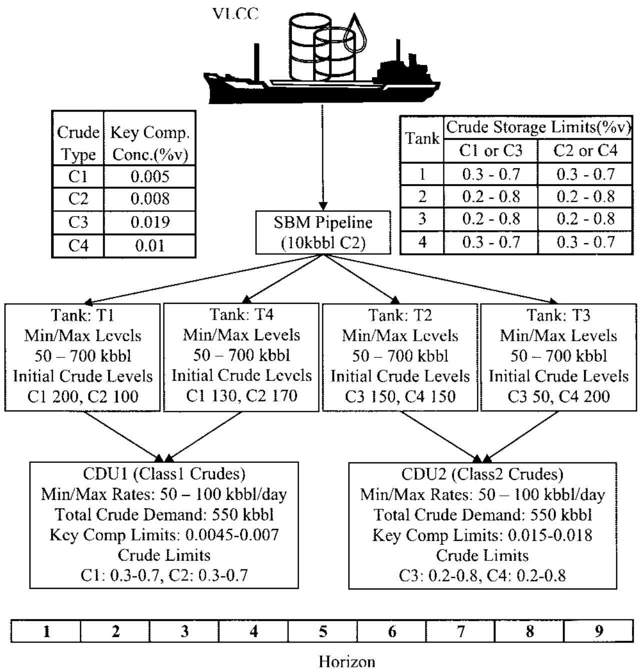

Figure 2. Operation schedule for the motivating example.

volumes in a VLCC are much larger than the SBM pipeline holdup.

(5) Crude mixing is perfect in each tank and time to change over tanks between processing units is negligible.

(6) For simplicity, only one key component decides the quality of a crude feed to CDU.

However, our approach can easily handle more key components and other quality constraints on crude oil cuts, as described later in the article.

Motivating Examples

We consider three examples to (1) provide some insight into the scheduling problem, (2) illustrate its complexity, and (3) highlight the issues that previous work has failed to resolve. As the first example, consider a refinery (Figure 2) with four storage tanks (T1, T2, T3, and T4), two CDUs (CDU1, CDU2), and one SBM line. The refinery handles four crudes (C1, C2, C3, and C4) and segregates them into two classes (Class1, Class2). As shown in Figure 2, C1 and C2 belong to Class1; C3 and C4 belong to Class2; T1, T4, and CDU1 handle Class1 crudes; and T2, T3, and CDU2 handle Class2 crudes. The scheduling horizon is 9 days and one VLCC carrying three parcels (300 kbbl C1, 300 kbbl C4, and 350 kbbl C3, unloaded in that sequence) arrives at the start of the horizon. The SBM pipeline holds 10 kbbl of C2 initially. Acceptable concentration (% volume) limits of the key component are [0.0045–0.007] for CDU1 and [0.015–0.018] for CDU2. Both CDU1 and CDU2 must process 550 kbbl of crude during the horizon. Figure 2 lists the minimum, maximum, and initial inventory levels in tanks, and initial levels and acceptable fractions of crudes. It also lists the acceptable limits on crude fractions in the feeds to CDUs. We assume that the schedule is in 1-day periods and the objective is to maximize the gross profit, which is the difference between the crude margins and the logistics costs such as sea-waiting cost (or demurrage) of VLCC, crude-mix changeover losses, and safety-stock penalty. Sea-waiting cost is \$10K per period (1 day), changeover cost is \$5K per occurrence, and penalty is \$0.2K per kbbl per period for depleting the total inventory beyond the minimum stock of 1200 kbbl. The crude margins are \$3K, \$4.5K, \$2K, and \$4K per kbbl of C1, C2, C3, and C4, respectively. We define crude margin as the total value of crude cuts from a crude oil (not the final refinery products) minus the cost for purchasing, transporting, and processing the crude.

In many refineries, crude scheduling is a largely manual task with little optimization. It is clear that a manual scheduling approach cannot effectively handle a complex scheduling objective such as the one mentioned above. It is almost impossible for the human scheduler to identify "optimal" crude mixes to process in each CDU. Therefore, such an approach would normally aim to unload the parcels as early as possible and then attempt to maintain constant feed rates to the CDUs, while minimizing crude-mix changeovers. Table 1 shows a candidate schedule obtained from such a strategy and the optimal schedule. Table 2 compares the profits of the two schedules. In spite of having an extra changeover, the optimal schedule reduces safety-stock penalty, but more important, increases the profit by using crude mixes in CDU2, which are more profitable. Even for this small example, rigorous optimization increases the gross profit by 3.1%. In the face of narrow margins and intense competition, this can make or break a refinery's bottom line.

Table 1. Candidate and Optimal Schedules for the Motivating Example

		Crude Amount [to CDU No.] (from Vessel No) in kbbl for Period*								
Schedule	Tank	1	2	3	4	5	6	7	8	9
1	1	−50[1]	−50[1]	−14.3[1]	−14.3[1]	−14.3[1]	−14.3[1]	−14.3[1]	−14.3[1]	−14.3[1]
(Manual)	2	−50[2]	+300(3) +100(4)		−50[2]	−50[2]	−75[2]	−75[2]	−75[2]	−75[2]
	3		−50[2]	−50[2]	+240(4)					
	4	+10(1) +300(2)		−50[1]	−50[1]	−50[1]	−50[1]	−50[1]	−50[1]	−50[1]
2	1	−50[1]	−50[1]	−8.33[1]	−8.33[1]	−8.33[1]	−8.33[1]	−8.33[1]	−8.33[1]	−100[1]
(Optimal)	2	−13.3[2]	+300(3)		−100[2]	−50[2]	−50[2]	−50[2]	−50[2]	−100[2]
	3	−36.7[2]	−50[2]	−50[2]	+340(4)					
	4	+10(1) +300(2)		−50[1]	−50[1]	−50[1]	−50[1]	−50[1]	−50[1]	

*"−" sign represents delivery to [CDU]; "+" sign represents receipt from (parcel).

Table 2. **Profit Comparison for Candidate and Optimal Schedules**

Profit Component	Manual (k$)	Optimal (k$)
Sea-waiting cost	0.00	0.00
Changeover cost	**15.00**	20.00
Safety stock penalty	52.14	**40.00**
Margin on CDU1	1905.27	1907.79
Margin on CDU2	1833.08	1936.55
Gross profit	3671.21	3784.33

Besides the above example, previous work (Kelley and Mann, 2003a,b; Lee et al., 1996; Li et al., 2002) has also clearly established the benefits of optimized crude scheduling. These benefits increase dramatically for larger and more complex systems. As mentioned by Lee et al. (1996) and Kelley and Mann (2003a,b), in addition to the economic benefits, automated crude scheduling results in more consistent schedules and helps even in the absence of skilled schedulers. Therefore, there is a clear incentive for developing systematic techniques to handle this scheduling problem.

The next two examples show that previous attempts at this problem have not fully succeeded. To this end, we first consider an example from Lee et al. (1996), to show that composition discrepancy can easily arise in their linearized formulation. Figure 3 gives the data for the example. Lee et al. (1996) solved it for the allowable key component concentration limits of 0.015–0.025 for Mix1 and 0.045–0.055 for Mix2. Let us refer to this as Case1. For Case1, Table 3 shows the key component concentration in the CDU feed for different periods, as obtained by solving Lee et al.'s MILP. By sheer coincidence, we observe no concentration discrepancy in this solution; that is, composition of crude sent from a tank matches that received by a CDU. However, let us see what happens when we change the problem specifications slightly. For instance, consider Case2, in which we relax the allowable key component concentration limits to 0.01–0.04 for Mix1 and 0.045–0.06 for Mix2. Table 3 gives the results from the MILP of Lee et al. (1996). During period 3, the crude in Mix1 has a key component concentration of 0.032. However, as we see from Table 3, the feed from Mix1 to the CDU has a key component concentration of 0.024 in period 3. The same happens during periods 5 to 8 in the case of Mix2. We can also see that a discrepancy also exists in the amounts of individual crude feeds to the CDU. For instance, Table 3 shows that the CDU should receive 28 kbbl of crude A and 22 kbbl of crude B in both periods 2 and 3. Instead, it receives 20 kbbl of A and 30 kbbl of B in period 2 and 36 kbbl of A and 14 kbbl of B in period 3. Thus, in the MILP of Lee et al. (1996), the crude composition in a feed tank may not match that in its feed to the CDU. Furthermore, it is clear from Table 3 that the key component and crude concentrations in the feed vary from period to period, even when they should remain unchanged. For instance, the key component concentrations are 0.051, 0.052, 0.055, and 0.060 for periods 4 to 7, respectively, for Mix2 feed. Similarly, as discussed above, the delivered crude amounts are different in periods 2 and 3 for Mix1 feed. In short, the MILP formulation of Lee et al. (1996) may lead to the above two forms of concentration discrepancy.

The reason for the discrepancies in the solutions proposed by Lee et al. is evident. As discussed later in our formulation, amounts of individual crudes that a charging tank feeds must be in proportion to its composition. When the tank composition is unknown, the constraints that enforce this requirement become bilinear. A discrete-time formulation (e.g., that of Lee et al., 1996) that approximates these bilinear constraints by linear ones will manifest the two discrepancies described above. This is explained by the fact that, because when the individual crude amounts (fed to CDU and held in tank) appear in linear constraints only, the optimizer is free to feed individual crudes without regard to their amounts in the tank; and, if there are no constraints that ensure that the tank composition remains constant, when the tank receives no crudes, then the crude amounts also vary arbitrarily over periods because they are free to do so. As rightly pointed out by Li et al. (2002), bilinear terms in the mass balance constraints can be replaced by exact linear constraints involving individual crude flows only for a continuous system with no mass accumulation.

Li et al. (2002) noted the first discrepancy and proposed an iterative MILP/NLP approach to correct it. In their algorithm, they first solve a MILP similar to that proposed by Lee et al. (1996), whose solution may have the concentration discrepancies. Based on that solution, they fix the vessel-to-tank and tank-to-CDU allocation variables and replace the linearized crude blending constraints by the exact nonlinear ones. This results in a NLP model, which they solve to correct the compositions. Then, they use these compositions in the MILP and resolve the MILP with correct linear composition constraints. Thus, they alternately solve MILP and NLP, until they satisfy some termination criteria. We used the same approach to solve the first motivating example (Figure 2) discussed earlier. Table 4 shows the crude amounts fed to the two CDUs during various periods, as obtained by solving the first MILP. It is clear that the delivered amounts vary from period to period and do not

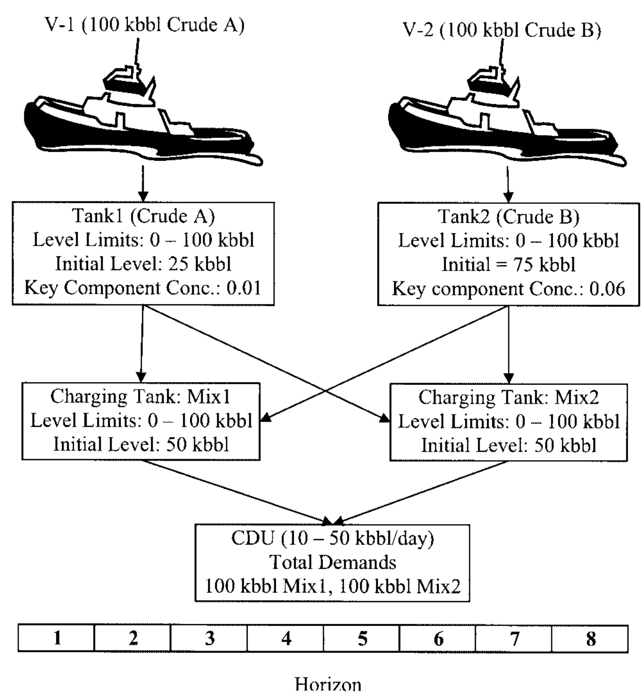

Figure 3. Operation schedule for the motivating example of Lee et al. (1996).

Table 3. Key Component Concentration in the Feed to CDU for Different Allowable Concentration Limits (Cases 1 and 2) for Charging Tanks in Lee et al. (1996) Motivating Example and Feed Composition Discrepancy in Case 2

Case	Charging Tank/Feed	Concentration of Key Component in Period $t = 0$	1	2	3	4	5	6	7	8
1 (Lee et al., 1996)	Mix 1 feed	0.02	0.02			0.025	0.025	0.025	0.025	0.025
	Mix 1	0.02	0.02			0.025	0.025	0.025	0.025	0.025
	Mix 2 feed	0.05	0.05	0.055	0.055					
	Mix 2	0.05	0.05	0.055	0.055					
2 (Relaxed limits on charging tanks)	Mix 1 feed	0.02		0.032	0.024					
	Mix 1	0.02		0.032	0.032					
	Mix 2 feed	0.05	0.05			0.051	0.052	0.055	0.06	0.06
	Mix 2	0.05	0.05			0.051	0.051	0.051	0.051	0.051
	Mix Tank.Crude Type	**Actual(Target) Crude Flow (kbbl) to CDU in Period* 0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	Mix1.A			20(28)	36(28)					
	Mix1.B			30(22)	14(22)					
	Mix2.A		10(10)			3(1.8)	3(1.8)	3(1.8)	(1.8)	(1.8)
	Mix2.B		40(40)			7(8.2)	7(8.2)	7(8.2)	10(8.2)	10(8.2)

*Target represents the crude flow that should be delivered as per tank composition.

respect the crude compositions in the tanks. Now, to correct this discrepancy, we must solve a NLP by fixing the allocations given by the first MILP and using the correct composition constraints. This means that the assignments of tanks for receipt and delivery are fixed for each period. To our surprise, we find that the resulting NLP is infeasible. A closer look reveals that T4 feeds CDU1 exclusively in periods 6–9. Its composition is 29.5% C2 and 70.5% C1, which does not meet the quality requirement (min 30% C2) for CDU1. To satisfy this quality requirement, both T1 and T4 must feed CDU1 in some proportion. However, the NLP of Li et al. (2002) has no freedom to change the tank-to-CDU allocations and thus cannot find a feasible solution. This clearly demonstrates that Li et al.'s algorithm may not find a solution in all cases, and reinforces the assertion of Kelly and Mann (2003a,b) that a decomposition-based heuristic strategy may fail to give a feasible solution, even when one exists. Furthermore, a global optimal solution to the NLP cannot be guaranteed in Li et al.'s algorithm.

The above discussion points to the need for an improved methodology for solving the crude scheduling problem. In this article, we present a novel discrete-time MILP formulation and a novel solution algorithm. Most past attempts at this problem have also used discrete representation of time. Pinto et al. (2000) suggested that, although continuous-time models substantially reduce the combinatorial complexity, discrete-time models are still attractive, in that they easily handle resource constraints and provide tighter formulations. In addition, we opine that discrete-time models offer several advantages. First, if we can successfully use slots of 8-h duration in a discrete-time formulation, as we have done in this article, then the complexity of a continuous-time formulation is not necessary. This is because most refiners prefer to begin their major operations at the start of a shift. Second, an effective approach for handling the bilinear constraints arising from the blending and accumulation of crudes in the storage tanks is still missing in the literature. Our use of a discrete-time formulation has enabled us to deal effectively with the inherent nonlinearity of this problem without solving a single NLP. Third, although our formulation uses discrete-time representation, it incorporates key features of a continuous-time formulation and largely obviates the need for a continuous-time model. Finally, our algorithm guarantees a solution that is free of all the drawbacks discussed earlier.

Table 4. Charging Schedule of CDUs Obtained Using Li et al. (2002) Approach for the Motivating Example

Tank	CDU	Crude	Initial Inventory (kbbl)[fraction]	Actual(Target) Crude Flow (kbbl) to CDU in Period or Crude{receipt} [Fraction] in Tank at Period End* 1	2	3	4	5	6	7	8	9
T1	1	C1	200[0.667]	35(33.35)	20(33.35)	35(33.25)	35(33.35)	35(33.35)				
	1	C2	100[0.333]	15(16.65)	30(16.65)	15(16.65)	15(16.65)	15(16.65)				
T2	2	C3	150[0.5]	10(25)	[0.208]		10(10.4)	10(10.4)	10(10.4)	10(0.4)	20(20.8)	20(20.8)
	2	C4	150[0.5]	40(25)	{300} [0.792]		40(39.6)	40(39.6)	40(39.6)	40(39.6)	80(79.2)	80(79.2)
T3	3	C3	50[0.2]		10(10)	10(10)	{340} [0.245]					
	3	C4	200[0.8]		40(40)	40(40)	[0.755]					
T4	4	C1	130[0.433]	{300} [0.705]					35(35.25)	35(35.25)	42(70.5)	70(70.5)
	4	C2	170[0.567]	{10} [0.295]					15(14.75)	15(14.75)	58(29.5)	30(29.5)

*Target represents the crude flow that should be delivered as per tank composition.

Unloading by SBM Line

We use a uniform discrete-time representation. Let NT identical periods ($t = 1, 2, \ldots, NT$) make up the scheduling horizon. In our opinion, periods shorter than 8 h are overkill for the present problem. The best choice is 8-h periods because most refineries operate in 8-h shifts and operators generally prefer to synchronize the starts of critical tasks with the starts of shifts. Shorter periods can give greater accuracy, but would excessively increase the computational burden. As the first step in our formulation, we break up all arriving multiparcel VLCCs into individual single-crude parcels.

Parcel creation

Before a VLCC can unload its first parcel, it must first eject the crude residing in the SBM pipeline. This ejected crude will normally not match the crude in the first parcel; thus we must treat it as a distinct single-crude parcel and it must transfer (to a tank) before the first parcel from the VLCC. By the same logic, the last parcel of a VLCC cannot transfer fully, because a portion equal to the SBM line capacity must remain in the SBM line and cannot transfer until the next VLCC starts unloading. As explained above, we must treat that portion as a distinct single-crude parcel; we call it an SBM parcel. In other words, unloading of each VLCC results in an extra single-crude SBM parcel from its last parcel. This obviously reduces the size of the last parcel in the VLCC. This gives us an ordered list (order of unloading) of all single-crude parcels, those in the VLCCs and their resulting SBM parcels. The first parcel in this list is an SBM parcel that originated from the last VLCC that visited the refinery in the past scheduling horizon. All parcels of the first VLCC to visit the refinery in the scheduling horizon follow next, in the order of their unloading. The last VLCC parcel will have a reduced size, but the SBM parcel that it created will follow next. This continues for all VLCCs. At the end of the current scheduling horizon, the SBM pipeline will hold the SBM parcel originating from the last parcel of the last VLCC to visit the refinery in the current scheduling horizon. Our list of parcels for the current scheduling exercise excludes this parcel. To illustrate this parcel creation step, consider a simple example with two VLCCs as in Figure 4. VLCC-1 has three parcels: 250 kbbl Oman, 300 kbbl Murban, and 110 kbbl Ratawi, which are to unload in that sequence. VLCC-2 also has three parcels: 250 kbbl Escravos, 250 kbbl Forcados, and 250 kbbl Arabmix, which are to unload in that sequence. At start, the SBM line holds 10 kbbl Kuwait. This is different from Oman in the first parcel of VLCC-1, so we must treat 10 kbbl Kuwait in the SBM line as a distinct parcel that must unload first. Thus, the first three parcels in our list become 10 kbbl Kuwait, 250 kbbl Oman, and 300 kbbl Murban. The Ratawi parcel of VLCC-1 will be fourth in the list, but with a reduced size of 100 kbbl, because 10 kbbl of Ratawi will remain in the SBM line, until VLCC-2 ejects it. Thus, the remaining parcels (the fourth and later) in the list become 100 kbbl Ratawi, 10 kbbl Ratawi, 250 kbbl Escravos, 250 kbbl Forcados, and 240 kbbl Arabmix. Note that the last parcel (Arabmix of VLCC-2) has a reduced size. The SBM parcel emanating from the Arabmix parcel will be first in the parcel list for the next scheduling horizon. Thus, we have two extra SBM parcels in our list to effectively model the operation of an SBM line with nonzero holdup.

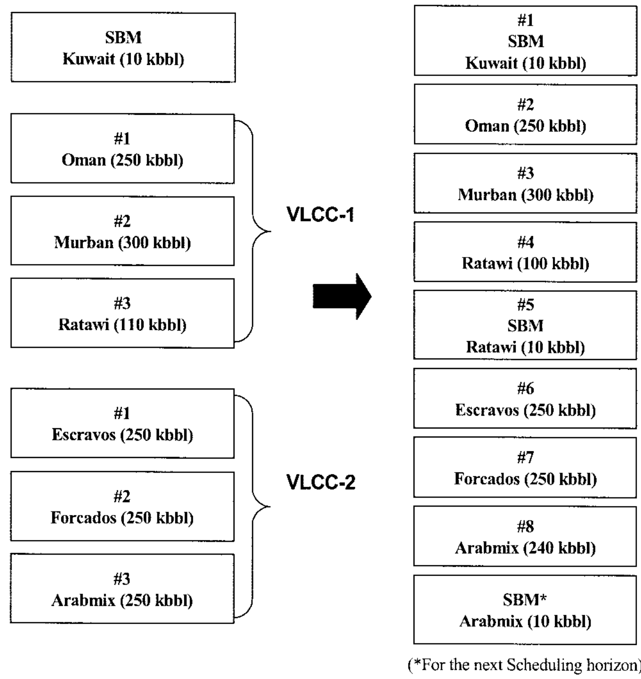

Figure 4. Parcel creation.

At the end of the parcel creation step, let there be NP parcels ($p = 1, 2, \ldots, NP$) in the list, which will unload exactly in the order in which they appear in the list. We now assign an arrival time ETA_p to parcel p as follows. ETA_p for a VLCC parcel is the arrival time of its VLCC, whereas that for an SBM parcel is the arrival time of the next VLCC.

The above procedure implicitly assumed that the size of the SBM line is far smaller than a typical parcel size. In this case, the SBM line will always have only one parcel at any time the crude is not flowing through the line. To extend the above procedure to handle the situation where the SBM line is a long-distance pipeline with a large holdup (comparable to or bigger than a typical parcel size), we simply need to allow for the possibility of multiple parcels in the line. Knowing the sequence in which other parcels move through the line, a slight modification of the above procedure will suffice for a large-holdup pipeline.

Having defined the periods and parcels, we are now in a position to develop the constraints in our MILP formulation. We begin with the parcel unloading operations.

Parcel-to-SBM connections

The SBM operation demands that each parcel connect to the SBM line to unload and then disconnect after unloading. To model this process of connection/disconnection, we define three binary variables

$$XP_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ is connected to the SBM line} \\ & \text{for unloading during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$XF_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ first connects to the} \\ & \text{SBM line at the start of period } t \\ 0 & \text{otherwise} \end{cases}$$

$$XL_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ disconnects from the SBM line at the end of period } t \\ 0 & \text{otherwise} \end{cases}$$

Based on ETA_p, we can identify the periods in which a parcel p can possibly be connected to the SBM line. Thus, we define XP_{pt}, XF_{pt}, and XL_{pt} only for $(p, t) \in \boldsymbol{PT} = \{(p, t) \mid \text{parcel } p$ may be connected to the SBM line in $t\}$. The following constraints relate these variables

$$XP_{pt} = XP_{p(t-1)} + XF_{pt} - XL_{p(t-1)} \qquad (p, t) \in \boldsymbol{PT} \quad (1a)$$

$$XP_{pt} \geq XL_{pt} \qquad (p, t) \in \boldsymbol{PT} \quad (1b)$$

Now, we assume that each parcel connects to and disconnects from the SBM line once and only once, so

$$\sum_t XF_{pt} = \sum_t XL_{pt} = 1 \qquad (p, t) \in \boldsymbol{PT} \quad (2a,b)$$

Equations 1a, 1b, 2a, and 2b together ensure that XF_{pt} and XL_{pt} will be binary automatically, when the XP_{pt} are so. Therefore, we treat XF_{pt} and XL_{pt} as continuous variables. Using these variables, the time TF_p at which p connects and the time TL_p at which it disconnects are

$$TF_p = \sum_t (t-1) XF_{pt} \qquad (p, t) \in \boldsymbol{PT} \quad (3a)$$

$$TL_p = \sum_t tXL_{pt} \qquad (p, t) \in \boldsymbol{PT} \quad (3b)$$

As we indicate later in detail, Eqs. 1–3 represent a novel approach for dealing with parcel unloading, which uses much fewer binary variables than other approaches (Lee et al., 1996; Li et al., 2002) in the literature and gives full flexibility.

Although two parcels cannot connect to the SBM line at a given *instance*, a parcel can disconnect and the next parcel connects during a period t. This would help to fully utilize the time available in a period in a discrete-time formulation and embed some continuous-time features in such a formulation. In principle, several small parcels can connect and disconnect in this manner in a given period, but for simplicity, we allow at most two parcels to connect in one period by using

$$\sum_p XP_{pt} \leq 2 \qquad (p, t) \in \boldsymbol{PT} \quad (4)$$

$$TF_{(p+1)} \geq TL_p - 1 \quad (5)$$

Equation 5 ensures that two parcels can connect in one period, only if the first of them disconnects in that period.

Finally, a parcel can unload only after its arrival time; therefore

$$TF_p \geq ETA_p \quad (6)$$

SBM-to-tank connections

As mentioned earlier, most refineries segregate crudes. To effect this crude segregation, we define a set $\boldsymbol{PI} = \{(p, i) \mid \text{tank}$ i may receive crude from parcel $p\}$. Now, to receive crude from a parcel, a tank must also connect to the SBM line. To model this process, we use

$$XT_{it} = \begin{cases} 1 & \text{if tank } i \text{ is connected to the SBM line during period } t \\ 0 & \text{otherwise} \end{cases}$$

Because we allowed (Eq. 4) at most two parcels to connect to the SBM line in one period, we do the same for tank-to-SBM connections

$$\sum_i XT_{it} \leq 2 \quad (7)$$

Clearly, a tank i cannot receive crude from a parcel during a period, unless both the parcel and tank are connected to the SBM line during that period. Therefore, we define a 0–1 continuous variable $X_{pit} = XP_{pt}XT_{it}$, which is one, only when both parcel p and tank i are connected to the SBM line during t. We linearize this nonlinear relation by using

$$X_{pit} \geq XP_{pt} + XT_{it} - 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \quad (8)$$

$$\sum_i X_{pit} \leq 2XP_{pt} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \quad (9a)$$

$$\sum_p X_{pit} \leq 2XT_{it} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \quad (9b)$$

We summed both sides of $X_{pit} = XP_{pt}XT_{it}$ over i and p and used Eqs. 4 and 7 to obtain Eqs. 9a and 9b. The above constraints ensure that X_{pit} will be binary, when XP_{pt} and XT_{it} are so. Note that this allocation variable was defined as a tri-index binary variable by Lee et al. (1996). Equations 4 and 7 admit at most four sequential tank-parcel connections in one period. To restrict such connections to at most two, we further impose

$$\sum_p \sum_i X_{pit} \leq 2 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \quad (10)$$

For most cases, Eqs. 4, 7, 9a, 9b, and 10 should suffice. However, if one wants to allow more parcel-to-tank transfers in a period, then one can generalize these equations as follows. To allow M parcel-to-SBM and N tank-to-SBM connections, and L parcel-to-tank connections in a period, we use

$$\sum_p XP_{pt} \leq M \qquad (p, t) \in \boldsymbol{PT} \quad (4i)$$

$$\sum_i XT_{it} \leq N \quad (7a)$$

$$\sum_{i} X_{pit} \le NXP_{pt} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \tag{9c}$$

$$\sum_{p} X_{pit} \le MXT_{it} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \tag{9d}$$

$$\sum_{p} \sum_{i} X_{pit} \le L \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \tag{10i}$$

Tank-to-CDU connections

For supplying its crude for processing, a tank must connect to one or more CDUs. We model this connection by the following binary variable

$$Y_{iut} = \begin{cases} 1 & \text{if tank } i \text{ feeds CDU } u \text{ during period } t \\ 0 & \text{otherwise} \end{cases}$$

Operating policies may dictate that a tank may not charge more than some (say two) CDUs simultaneously and vice versa. Thus

$$\sum_{u} Y_{iut} \le 2 \qquad (i, u) \in \boldsymbol{IU} \tag{11a}$$

$$\sum_{i} Y_{iut} \le 2 \qquad (i, u) \in \boldsymbol{IU} \tag{11b}$$

where $\boldsymbol{IU} = \{(i, u) \mid \text{tank } i \text{ can feed CDU } u\}$. Similarly, most often in practice, a tank cannot feed a CDU, when it is connected to the SBM line or is settling brine after receiving crude. Assuming a brine settling time of 8 h or one period, we use

$$2XT_{it} + Y_{iut} + Y_{iu(t+1)} \le 2 \qquad (i, u) \in \boldsymbol{IU} \tag{12}$$

Crude delivery and processing

Having modeled the parcel-to-SBM, SBM-to-tank, and tank-to-CDU connections, we are ready to transfer crude between tanks and parcels, and tanks and CDUs. We first consider the transfers from parcels to tanks. To this end, we define FPT_{pit} as the amount of crude transferred from parcel p to tank i during period t. This transfer can occur only when both tank i and parcel p are connected to the SBM line, and its amount must satisfy some upper limit fixed by the maximum pumping rate of crude to tank i

$$FPT_{pit} \le FPT_{pi}^{U} X_{pit} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \tag{13}$$

In the event that multiple sequential transfers occur in the same period, the total time required for all transfers must not exceed the period length; therefore

$$\sum_{p} \sum_{i} \frac{FPT_{pit}}{FPT_{pi}^{U}} \le 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \tag{14}$$

Finally, each parcel p must unload fully during the scheduling horizon, so if PS_p denotes the size of parcel p, then

$$\sum_{i,t} FPT_{pit} = PS_{p} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \tag{15}$$

As with tanks, most refineries also segregate CDUs. Therefore, to see whether a tank can feed a CDU, we define sets $\boldsymbol{IU} = \{(i, u) \mid \text{tank } i \text{ can feed CDU } u\}$ and $\boldsymbol{IC} = \{(i, c) \mid \text{tank } i \text{ may have crude } c \text{ sometime during the horizon}\}$. For delivering crude to CDUs, we define $FCTU_{iuct}$ as the amount of crude c delivered by tank i to CDU u during period t. Then, the total amount FTU_{iut} of crude that tank i feeds to CDU u during t is

$$FTU_{iut} = \sum_{(i,c) \in IC} FCTU_{iuct} \qquad (i, u) \in \boldsymbol{IU} \tag{16}$$

The above amount can be nonzero only when tank i is connected to CDU u during t, and it must satisfy some lower and upper limits

$$Y_{iut} FTU_{iu}^{L} \le FTU_{iut} \le Y_{iut} FTU_{iu}^{U} \qquad (i, u) \in \boldsymbol{IU} \tag{17}$$

Because multiple tanks may feed one CDU, the total feed FU_{ut} to CDU u during t is

$$FU_{ut} = \sum_{(i,u) \in IU} FTU_{iut} \tag{18}$$

This must be within the processing limits of CDU u, so

$$FU_{ut}^{L} \le FU_{ut} \le FU_{ut}^{U} \tag{19}$$

In practice, the plant operation may know that its CDUs cannot process crude mixtures with some extreme fractions of crudes. Therefore, when crude storage facility is the bottleneck, many refiners use rough heuristics to control the crude quality in storage tanks to prevent unmanageable crude blends. To implement such heuristics, we use

$$FU_{ut} xc_{cu}^{L} \le \sum_{i} FCTU_{iuct} \le FU_{ut} xc_{cu}^{U} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \tag{20}$$

where xc_{cu}^{L} and xc_{cu}^{U} are the allowable lower and upper limits on the fraction of crude c in the feed to CDU u. Clearly, one can easily relax these acceptable fractions of crude limits, as needed. Similarly, to avoid processing problems in CDUs and other downstream units, the plant operation may wish to keep the concentration of some key component such as sulfur or metal, for example, below certain limits for a CDU. Crude assays normally include concentrations of such key components. If xk_{kc} represents this known fraction (not a variable) of key component k in crude c, then we ensure feed quality by using

$$xk_{ku}^{L} FU_{ut} \le \sum_{i} \sum_{c} FCTU_{iuct} xk_{kc} \le xk_{ku}^{U} FU_{ut} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \tag{21}$$

Note that the above can be duplicated for any other key component in an appropriate form.

In real operation, one would want to minimize the upsets caused by changeovers of tanks (thus crudes) feeding to a CDU. To detect such changes, we define a 0–1 continuous variable $YY_{iut} = Y_{iut}Y_{iu(t+1)}$, which is one if tank i is connected to CDU u during both periods t and $(t + 1)$. We linearize YY_{iut} as follows

$$YY_{iut} \geq Y_{iut} + Y_{iu(t+1)} - 1 \qquad (i, u) \in \boldsymbol{IU} \tag{22a}$$

$$YY_{iut} \leq Y_{iu(t+1)} \qquad (i, u) \in \boldsymbol{IU} \tag{22b}$$

$$YY_{iut} \leq Y_{iut} \qquad (i, u) \in \boldsymbol{IU} \tag{22c}$$

Then, for detecting the presence of a changeover on a CDU, we use

$$CO_{ut} \geq Y_{iut} + Y_{iu(t+1)} - 2YY_{iut} \qquad (i, u) \in \boldsymbol{IU} \tag{23}$$

When multiple tanks feed a CDU, the composition of feed can change simply because of a change in the feed rates from various tanks. This would upset the CDU operation. However, if only a single tank is feeding a CDU, then a change in its feed rate would not upset the CDU. Therefore, to prohibit a change in composition, when two tanks are feeding a CDU, we force the feed flow rates of individual tanks to remain constant by using the following where M is a large number

$$M\left[2 - \sum_i YY_{iut}\right] + FTU_{iut} \geq FTU_{iu(t+1)} \qquad (i, u) \in \boldsymbol{IU} \tag{24a}$$

$$M\left[2 - \sum_i YY_{iut}\right] + FTU_{iu(t+1)} \geq FTU_{iut} \qquad (i, u) \in \boldsymbol{IU} \tag{24b}$$

Crude inventory

First, to identify the crude in parcel p, we define a set $\boldsymbol{PC} = \{(p, c) \mid \text{parcel } p \text{ carries crude } c\}$. Using VCT_{ict} to denote the amount of crude c in tank i at the end of period t, we obtain the following from a crude balance on tank i

$$VCT_{ict} = VCT_{ic(t-1)} + \sum_{(p,c)\in\boldsymbol{PC},(p,t)\in\boldsymbol{PT}} FPT_{pit} - \sum_{(i,u)\in\boldsymbol{IU}} FCTU_{iuct} \qquad (i, c) \in \boldsymbol{IC} \tag{25}$$

With this, the total crude level in tank i at the end of period t is given by

$$V_{it} = \sum_{(i,c)\in\boldsymbol{IC}} VCT_{ict} \tag{26}$$

This must satisfy some upper and lower limits as

$$V_i^L \leq V_{it} \leq V_i^U \tag{27}$$

Because of processing and operational constraints, crude fractions in tanks may be kept in some limits as follows

$$xt_{ic}^L V_{it} \leq VCT_{ict} \leq xt_{ic}^U V_{it} \tag{28}$$

Crude is normally stored in floating roof tanks to minimize evaporation losses. Such a tank requires a minimum crude level (or heel) to avoid damage to the roof, when the tank goes empty. Because of the presence of heel, crudes usually accumulate in the tank over time. However, a crude type with negligible volume fraction does not significantly affect the overall quality. Thus, to limit the number of crudes in a tank, it is advisable to retain only the crudes with significant volume fractions and normalize their initial fractions in the tank accordingly. Recall that the number of crudes in a tank affects the problem size.

When a tank i feeds a CDU u, the amounts ($FCTU_{iuct}$) of individual crudes c delivered must be in proportion to the crude composition in the tank. If $f_{ict} = VCT_{ict}/V_{it}$ denotes the volume fraction of crude c in tank i during period t, then the following must hold

$$FCTU_{iuct} = f_{ict}FTU_{iut} \tag{16a}$$

$$VCT_{ict} = f_{ict}V_{it} \tag{26a}$$

Except at the start of the scheduling horizon (or equivalently the first period), when we know the compositions of crudes in all tanks, we do not know f_{ict}. This makes f_{ict} variables (Eqs. 16a and 26a) bilinear, and the entire formulation MINLP.

Production requirements

We can specify them in several ways. One simple way is to specify a crude throughput demand for each CDU in each period as follows

$$FU_{ut} \geq D_{ut} \tag{29}$$

This obviously requires detailed data that may be difficult to obtain readily. A better way is to specify a throughput demand over the entire horizon for each CDU or groups of CDUs

$$\sum_i FU_{ut} = D_u \qquad \text{or} \qquad \sum_u \sum_t FU_{ut} = D \tag{30a,b}$$

To integrate the refinery supply chain operations, we may specify demands for the products rather than the crudes. Thus, if PD_j denotes the maximum demand for product j during the scheduling horizon, then

$$\sum_i \sum_u \sum_c \sum_t FCTU_{iuct} y_{jcu} \leq PD_j \tag{31}$$

where, y_{jcu} is the fractional yield of product j from crude c processed in CDU u.

Many final refinery products are produced by blending various crude oil cuts directly or after treatment or conversion. Most petroleum product blending is additive based on volume or weight. Metals such as sulfur, mercury, lead, arsenic, sodium, nickel, vanadium, and so forth in crudes are common impurities (key components) that lead to downstream processing difficulties. The distribution of these key components in different crude oil cuts is also important. Most of these tend to remain in heavier components, such as residue, heavy vacuum gas oil feed stock, for instance, and pose problems in conversion units (Resid crackers, hydrocrackers, FCCU, CCU, etc.). We can impose limits (as in Eq. 21) on the desired properties of various crude oil cuts and quantities of key components in them. We can also extend this to refinery products that are direct blends of crude oil cuts using various blending numbers or indices for properties such as pour points, flash point, Reid vapor pressure, and viscosities.

Scheduling objective

As mentioned earlier, one aim of short-term crude scheduling is to exploit the benefits of opportunistic crude mixes. We use the maximization of total gross profit as the scheduling objective instead of the minimization of operating cost because the former includes the effect of crude compositions and crude margins, whereas the latter does not. We define gross profit as the sum of crude margins (netbacks) minus the operating costs related to logistics.

As defined earlier in the motivating example, crude margin is the total value of cuts from a crude oil minus the costs of purchasing, transporting, and processing the crude. Note that crude margin does not include any costs related to the logistics of crude scheduling mentioned later. Because the product yields vary with crudes and CDUs, we define CP_{cu} as the margin ($ per unit volume) of crude c processed in CDU u.

The operating costs related to logistics are as follows. First, a change in the feed composition upsets the steady operation of a CDU. This is called a *changeover*. A changeover lasts a few hours, during which it perturbs the processing unit operation and leads to calling off special products such as ATF, for example; generation of off-spec products or slops; and additional work resulting in lost productivity. Clearly, every changeover incurs some cost to the refinery and is undesirable. Refiners strive to minimize the changeovers. Although quantifying the changeover losses exactly is difficult, one can approximately estimate the losses of undesirable effects mentioned above. We let COC denote the cost per changeover. The second cost in crude scheduling is the demurrage or sea-waiting cost. The logistics contract with each shipping vessel stipulates an acceptable sea-waiting period. If the vessel harbors beyond this stipulated time, then demurrage (or sea-waiting cost) incurs. We let SWC_v ($ per unit time) denote the demurrage or sea-waiting cost for VLCC v. Third, although unloading of crudes does incur costs, we exclude them from our scheduling objective. This is because the amount of crude imported is fixed for the scheduling horizon. Similarly, unlike previous work (Lee et al., 1996; Li et al., 2002), we also do not include the crude inventory cost because the refiner normally makes the purchasing decisions far in advance of scheduling and as such these decisions fall beyond the purview of the scheduling activity. Once the refiner purchases crude, it becomes an integral part of the system and incurs inventory cost irrespective of the scheduling. However, one inventory-related decision does fall under the purview of scheduling activity. That is the desire of most refiners to maintain a minimum stock of crude to guard against uncertainty. Let SS denote the desired safety stock of crude and SSP the penalty ($ per unit volume per period) for underrunning the crude safety stock. Based on the above discussion, we obtain the total gross profit as

$$\text{Profit} = \sum_i \sum_u \sum_c \sum_t FCTU_{iuct} CP_{cu} - \sum_v DC_v - COC \sum_u \sum_t CO_{ut} - \sum_t SC_t \quad (32)$$

$$DC_v \geq (TL_p - ETA_p - ETD_v) SWC_v \qquad (p, v) \in \boldsymbol{PV} \quad (33)$$

$$SC_t \geq SSP\left(SS - \sum_i V_{it}\right) \quad (34)$$

where ETD_v is the estimated time of departure of VLCC v as agreed in the logistics contract, $\boldsymbol{PV} = \{(p, v) \mid \text{parcel } p \text{ is the last parcel in VLCC } v\}$, DC_v is the demurrage cost for VLCC v, and SC_t is the stock penalty for period t.

This completes our formulation (Eqs. 1a to 34) for a refinery with one SBM pipeline. However, refineries often use jetties with or without an SBM. We need only slight modifications in the above formulation to allow crude unloading by jetties. To this end, we now present a formulation for a refinery with J identical jetties, but no SBM line.

Unloading by Jetties

Unloading by a jetty is analogous to unloading by an SBM line, except for some differences. We assume that only single-crude vessels berth at jetties, so we can treat a vessel berthing at a jetty as simply a single-parcel VLCC. In contrast to the SBM line, the holdup in a jetty-to-tank transfer line is small and its effect on the composition of the receiving tank negligible. Thus, we need not consider any new parcels (such as SBM parcels considered earlier) arising from this holdup. The connection/disconnection process of a vessel to a jetty is analogous to that of a parcel to the SBM except that we have J identical berths instead of one SBM station.

Based on the above discussion, it is clear that we can use all the variables in the SBM formulation with their usual meanings to handle jetties. Thus, in the ensuing formulation, we discuss only those constraints that are absent or different from those of the previous formulation.

First, to allow J vessels to berth and unload simultaneously, we must drop Eqs. 5 and 9 and modify Eqs. 4 and 10 as follows

$$\sum_p XP_{pt} \leq J \qquad (p, t) \in \boldsymbol{PT} \quad (4a)$$

$$\sum_p \sum_i X_{pit} \leq 2J \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \quad (10a)$$

We continue to use Eqs. 9a and 9b to ensure that a parcel can unload to at most two tanks in the same period and a tank can

Table 5. Constraints for Different Refinery Configurations

Refinery Configuration	Parcel-to-SBM/Jetty Connections	SBM/Jetty-to-Tank Connections	Tank-to-CDU Connections	Crude Delivery and Processing	Crude Inventory	Product Requirement	Scheduling Objective	Tank-to-Tank Transfers
	Constraint Equations for							
SBM Only	1a,b 2a,b, 3a,b, 4–6	7, 8, 9a,b, 10	11a,b, 12	13–21, 22a,b,c, 23, 24a,b	25–28	29, 30a,b, 31	32–34, 32a	25a, 35, 36a,b, 37–40, 41a,b, 42–44
Jetties Only	1a,b, 2a,b, 3a,b, 4a, 6	8, 9a,b, 10a	11a,b, 12	13, 14a,b, 15–21, 22a,b,c, 23, 24a,b	25–28	29, 30a,b, 31	32–34, 32a	25a, 35, 36a,b, 37–40, 41a,b, 42–44
SBM and Jetties	1a,b, 2a,b, 3a,b, [4, 5], (4a), 6	8, 9a,b, [7, 10], (10a)	11a,b, 12	13, 14a,c, 15–21, 22a,b,c, 23, 24a,b	25–28	29, 30a,b, 31	32–34, 32a	25a, 35, 36a,b, 37–40, 41a,b, 42–44

[Equations] are for SBM and VLCC parcels only, whereas (equations) are for jetty parcels only.

receive from at most two parcels in the same period. This is mainly for simplicity; if facilities permit, we can increase the number as suited. For multiple sequential transfers occurring in the same period, we replace Eq. 14 by

$$\sum_{i} \frac{FPT_{pit}}{FPT^{U}_{pi}} \le 1 \qquad (p,\, t) \in \boldsymbol{PT},\, (p,\, i) \in \boldsymbol{PI} \tag{14a}$$

$$\sum_{p} \frac{FPT_{pit}}{FPT^{U}_{pi}} \le 1 \qquad (p,\, t) \in \boldsymbol{PT},\, (p,\, i) \in \boldsymbol{PI} \tag{14b}$$

This completes the extension of our formulation to a refinery with J jetties and no SBM. Table 5 lists its required constraints. Having derived the separate formulations for SBM and jetties, we now combine them into one formulation for a refinery with both one SBM and J jetties.

Unloading by SBM and Jetties

In this case, both multiparcel VLCCs and single-parcel vessels will arrive at different times during the scheduling horizon. We treat all of them as vessels. In essence, we have two sets of vessels. One set of vessels unloads by the SBM line, whereas the other unloads by the jetties. After we create the SBM parcels for the VLCCs as explained earlier, we have three types of parcels; that is, SBM parcels, VLCC parcels, and jetty parcels. For convenience, we include the SBM parcels as the first parcels in the subsequent VLCCs. Thus, we now have only VLCC parcels and jetty parcels, and we use the appropriate constraints developed exclusively for each set. Let ***SP*** denote the set of VLCC parcels and ***JP*** denote the set of jetty parcels.

As discussed earlier, Eqs. 1a, 1b, 2a, 2b, 3a, 3b, 6, 8, 9a, and 9b hold for both parcel sets; Eqs. 4 and 5 and 7 and 10 hold for ***SP*** only; and Eqs. 4a and 10a hold for ***JP*** only. All other constraints are common to both ***SP*** and ***JP***, except those (Eqs. 14 and 14a and 14b) governing multiple sequential transfers within a period. Instead of Eqs. 14 and 14a,b, we use Eq. 14a for both ***SP*** and ***JP*** and add the following constraint

$$\sum_{p \in SP} \frac{FPT_{pit}}{FPT^{U}_{pi}} + \sum_{p \in JP} \frac{FPT_{pit}}{FPT^{U}_{pi}} \le 1 \tag{14c}$$

This completes the formulation for a refinery with both one SBM and J jetties, and we are now ready to address the practical feature of tank-to-tank transfers.

Tank-to-Tank Transfers

In practice, one may need to transfer crude from one tank to another to facilitate a quick crude receipt and avoid demurrage. To model these transfers, we use the following binary variable

$$Z_{ii't} = \begin{cases} 1 & \text{if a crude exchange occurs} \\ & \text{between tanks } i \text{ and } i' \text{ during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$i' > i,\, (i,\, i') \in \boldsymbol{II}$$

where $\boldsymbol{II} = \{(i,\, i') \mid \text{crude transfer between tank } i \text{ and } i' \text{ is allowed}\}$. Note that we defined $Z_{ii't}$ only for combinations of i and i' and not permutations. Furthermore, $Z_{ii't}$ defines only the existence of a transfer, not its direction. To identify the number of times a tank i exchanges crude with another tank in a given period, we define

$$ZT_{it} = \sum_{i'>i,(i,i')\in \boldsymbol{II}} Z_{ii't} + \sum_{i>i',(i,i')\in \boldsymbol{II}} Z_{i'it} \tag{35}$$

Tank-to-tank transfers do complicate operations and refiners use them only when no other option is possible. Therefore, we allow at most one tank-to-tank transfer in a period

$$ZT_{it} \le 1 \tag{36a}$$

$$\sum_{i} ZT_{it} \le 2 \tag{36b}$$

Note that ZT_{it} will automatically be binary, so we treat it as a continuous variable. Similarly, we restrict the total number of tank-to-tank transfers in the scheduling horizon to a small number m by using

$$\sum_t \sum_i ZT_{it} \leq 2m \tag{37}$$

As we did earlier (Eq. 12) with a tank receiving crude, we assume that a tank involved in a tank-to-tank transfer cannot feed a CDU. Therefore

$$ZT_{it} + Y_{iut} \leq 1 \qquad (i, u) \in \boldsymbol{IU} \tag{38}$$

So far, we addressed only the existence of a transfer, but neither its direction nor amount. To model the direction and amount, we define a continuous variable $FCTT_{ii'ct}$ as the amount of crude c transferred from tank i to tank i'. $FCTT_{ii'ct}$ is positive, when the transfer is from i to i', and vice versa. Of the two tanks engaged in a tank-to-tank transfer, one must deliver and the other must receive; therefore

$$FCTT_{ii'ct} + FCTT_{i'ict} = 0 \qquad (i, i') \in \boldsymbol{II} \tag{39}$$

With this, the total amount of a tank-to-tank transfer from i to i' in period t becomes

$$FTT_{ii't} = \sum_c FCTT_{ii'ct} \qquad (i, i') \in \boldsymbol{II} \tag{40}$$

To obtain the absolute amount $AFTT_{ii't}$ ($i' > i$) of a tank-to-tank crude transfer, we use

$$AFTT_{ii't} \geq FTT_{ii't} \qquad i' > i,\ (i, i') \in \boldsymbol{II} \tag{41a}$$

$$AFTT_{ii't} \geq FTT_{i'it} \qquad i' > i,\ (i, i') \in \boldsymbol{II} \tag{41b}$$

Clearly, $AFTT_{ii't}$ must be zero, when $Z_{ii't}$ is zero, and it must also have an upper limit, so

$$AFTT_{ii't} \leq FTT^U_{ii'}[\delta_{ii'}Z_{ii't} + (1 - \delta_{ii'})Z_{i'it}] \qquad i' > i,\ (i, i') \in \boldsymbol{II} \tag{42}$$

where $\delta_{ii'} = 1$ for $i' > i$ and zero otherwise, and $FTT^U_{i'i}$ denotes the maximum amount of tank-to-tank transfer possible between i and i' in a period. Having constrained the absolute transfer amount, we can now constrain the individual crude transfer amounts. We do this as follows

$$-AFTT_{ii't}\max[xt_{ic}, xt_{i'c}] \leq FCTT_{ii'ct} \leq AFTT_{ii't}\max[xt_{ic}, xt_{i'c}] \qquad i' > i,\ (i, i') \in \boldsymbol{II} \tag{43}$$

The above transfers modify the individual crude balance (Eq. 25) as

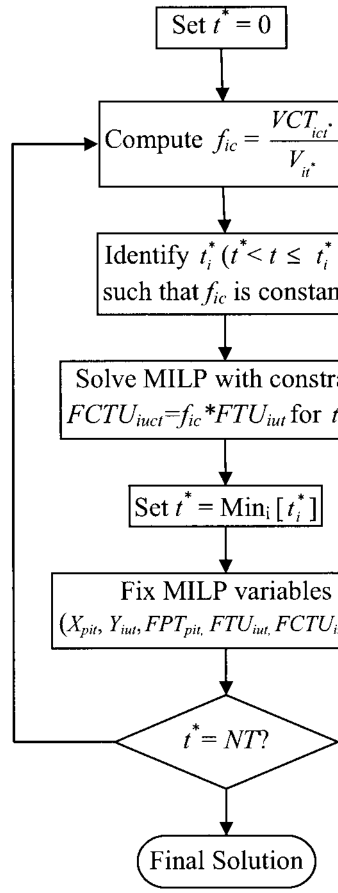

Figure 5. Flow chart for the solution algorithm.

$$VCT_{ict} = VCT_{ic(t-1)} + \sum_{\substack{(p,c)\in \boldsymbol{PC}\\(p,t)\in \boldsymbol{PT}}} FPT_{pit} - \sum_{(i,u)\in \boldsymbol{IU}} FCTU_{iuct} + \sum_{\substack{i'\neq i\\(i',c)\in \boldsymbol{IC}\\(i,i')\in \boldsymbol{II}}} FCTT_{i'ict} \qquad (i, c) \in \boldsymbol{IC} \tag{25a}$$

Finally, we allow a tank to receive crude from both a parcel and another tank in the same period, as long as the two transfers take place sequentially in the period. Therefore

$$\sum_{i'\neq i} \frac{AFTT_{ii't}}{FTT^U_{ii'}} + \sum_p \frac{FPT_{pit}}{FPT^U_{pi}} \leq 1 \qquad (i, i') \in \boldsymbol{II} \tag{44}$$

In brief, to allow tank-to-tank transfers in our formulation, we replace Eq. 25 by Eq. 25a in our SBM/jetty formulation and add Eqs. 35 to 44. Table 5 lists the required constraints for various formulations. Tank transfers increase the problem size and complexity drastically because of the additional decisions, variables, and constraints, making the problem compute intensive. Because the main goal of tank-to-tank transfers is to avoid demurrage, it may be desirable to allow them only in a few periods. For instance, one may allow them only in the period before the arrival and in periods during the berthing of a vessel. In such a case, we impose Eq. 25a only for such periods and Eq. 25 for the remaining periods.

We observed that it was possible to get a better schedule

Table 6. Details of Individual Iterations for the Motivating Example Using the Proposed Algorithm

Iteration	t^*	Tank	t_i^*	$\min_i[t_i^*]$	MILP Profit (k$)	Number of Binary Variables	CPU Time (s)	Composition f_{ic} C1 or C3	C2 or C4
0	0	T1	1	1	3879	53	2.719	0.667	0.333
		T2	2					0.500	0.500
		T3	2					0.200	0.800
		T4	1					0.433	0.567
1	1	T1	EOH*	2	3879	45	1.235	0.667	0.333
		T2	2					0.500	0.500
		T3	2					0.200	0.800
		T4	EOH					0.705	0.295
2	2	T1	EOH	3	3819.78	35	0.562	0.667	0.333
		T2	3					0.244	0.756
		T3	3					0.200	0.800
		T4	EOH					0.705	0.295
3	3	T1	EOH	4	3815.35	27	0.469	0.667	0.333
		T2	4					0.244	0.756
		T3	4					0.200	0.800
		T4	EOH					0.705	0.295
4	4	T1	EOH	EOH	3784.33	20	0.469	0.667	0.333
		T2	EOH					0.244	0.756
		T3	EOH					0.800	0.200
		T4	EOH					0.705	0.295

*EOH, end of horizon.

without tank transfers than with. To avoid such discrepancy, one can include a penalty or cost (*TTC*) in the objective function for each tank transfer operation. With this, the objective function becomes

$$\text{Profit} = \sum_i \sum_u \sum_c \sum_t FCTU_{iuct} CP_{cu} - \sum_v DC_v - COC \sum_u \sum_t CO_{ut} - \sum_t SC_t - \frac{TTC}{2} \sum_i \sum_t ZT_{it} \quad (32a)$$

The crude scheduling problem, as discussed here, is an inherently nonlinear problem as a result of Eqs. 16a and 26a. Because Lee et al. (1996) used a linear formulation without these bilinear constraints, their solutions suffered from composition discrepancies as described in the motivating examples. As discussed earlier, to avoid this discrepancy, Li et al. (2002) solved MILPs and NLPs, but their approach could not guarantee convergence in all cases. We now develop a novel iterative strategy that simply solves a series of relaxed MINLPs (or equivalently MILPs) of reducing size and complexity to obtain a near-optimal solution with no discrepancy in composition.

Solution Algorithm

To avoid MINLP/NLP solutions, we relax our MINLP by dropping Eqs. 16a and 26a. The resulting relaxation is a MILP, and will inevitably suffer from the composition discrepancy. This is because the constraints involving $FCTU_{iuct}$ are now all linear and the optimizer can push arbitrary amounts of individual crudes rather than the correct mixture to CDU. Clearly, it is not wise to neglect Eqs. 16a and 26a entirely, and we must use them in some form in our solution algorithm. Besides, Eqs. 16, 17, 20, 21, and 26 serve as their linear approximations.

The main idea behind our algorithm is as follows. First, observe that every tank has blocks of contiguous periods during which its composition does not change. For such a block, if we

Table 7. Tanker Arrival Details, Crude Demands, and Key Component Concentration Limits on CDUs for Examples 2 to 6

Ex	Tanker	Arrival Period	Parcel No.: (Crude, Parcel Size kbbl)	CDU	T Lower	Upper	Demand (kbbl)
2	VLCC-1	1	1: (C2, 10), 2: (C3, 250), 3: C4 (300), 4: C5 (100)	CDU1	0.001	0.0130	750
	VLCC-2	14	5: (C5, 10), 6: (C6, 250), 7: (C3, 250), 8: (C8, 240)	CDU2	0.001	0.0125	750
				CDU3	0.001	0.0035	750
3	VLCC-1	1	1: (C2, 10), 2: (C3, 350), 3: (C4, 200), 4: (C5, 300)	CDU1	0.001	0.0135	1000
	VLCC-2	16	5: (C5, 10), 6: (C6, 200), 7: (C8, 250), 8: (C3, 240)	CDU2	0.001	0.0130	1000
	VLCC-3	28	9: (C3, 10), 10: (C6, 250), 11: (C2, 250), 12: (C7, 190)	CDU3	0.001	0.0040	1000
4	V1-V2	3	1: (C2, 350), 2: (C3, 350)	CDU1	0.001	0.0130	500
	V3-V5	6	3: (C5, 350), 4: (C1, 300), 5: (C7, 350)	CDU2	0.001	0.0125	500
	V6	9	6: (C8, 250)	CDU3	0.001	0.0035	600
	V7-V8	10	7: (C3, 250), 8: (C6, 300)				
5	VLCC-1	3	1: (C2, 10), 2: (C6, 100), 3: (C1, 100), 4: (C4, 90)	CDU1	0.0125	0.0185	600
	V1-V2	5	5: (C2, 125), 6: (C5, 125)	CDU2	0.0125	0.0175	600
	V3	6	7: (C3, 100)	CDU3	0.004	0.0070	600
6	VLCC-1	2	1: (C2, 10), 2: (C4, 500), 3: (C3, 500), 4: (C5, 440)	CDU1	0.001	0.0140	375
				CDU2	0.001	0.0125	375
				CDU3	0.001	0.0030	400

Table 8. Detailed Problem Data for Examples 2 to 6

	Capacity (kbbl)		Heel (kbbl)		Initial Inventory (kbbl)		Initial Crude Composition (kbbl)						
							Example 2–4, [6]				Example 5		
Tank	Ex 2–4, 6	Ex 5	Ex 2–4, 6	Ex 5	Ex 2–4, [6]	Ex 5	C1 or 5	C2 or 6	C3 or 7	C4 or 8	C1 or 5	C2 or 6	C3 or 7
T1	570	400	60	50	350	250	50	100	150	50	100	100	50
T2	570	400	60	50	400	200	200	0	50	150	50	100	100
T3	570	400	60	50	350	300	100	100	50	100	100	100	100
T4	980	400	110	50	950	350	200	250	200	300	100	150	100
T5	980	400	110	50	300	250	100	100	50	50	100	75	75
T6	570	400	60	50	80[240]	100	20[30]	20[30]	20[150]	20[30]	25	25	50
T7	570	400	60	50	80[120]	100	20[30]	20[30]	20[50]	20[10]	50	25	25
T8	570	400	60	50	450[550]	250	100[150]	100	100[210]	150[90]	75	75	100

	Flow Rate Limits (kbbl/Period)												
Ex	Parcel–Tk Min–Max	Tk–CDU Min–Max	Tk–Tk Min–Max	Demurrage Cost (k$/period)	Changeover Loss (k$/instance)	Safe Inventory Penalty ($/bbl/period)	Crude	Key Comp. Conc. (fr) Ex 2–4	Ex 5	Ex 6	Margin ($/bbl) Ex 2–4	Ex 5	Ex 6
2 to 4	10–400	20–45(40*)		25	10	0.2	C1	0.002	0.005	0.0025	1.5	1.5	1.5
5	10–250	0–50		15	5	0.2	C2	0.0025	0.008	0.0025	1.7	1.75	1.7
6	10–400	0–45	0–400	50	25	0.2	C3	0.0015	0.004	0.004	1.5	1.85	1.5
							C4	0.006	0.015	0.002	1.6	1.25	1.6
							C5	0.012	0.01	0.01	1.45	1.45	1.45
							C6	0.013	0.02	0.015	1.6	1.65	1.6
							C7	0.009		0.014	1.55		1.55
							C8	0.015		0.011	1.6		1.6

*For example 3; Date in [ ] indicate the change in data for Ex 6 from Ex 2 to 4
#Tanks 1, 6–8 store crudes 1–4; 2–5 store 5–8 for Ex 2–4 and 6
*Tanks 1, 6–8 store crudes 1–3 (Class 1); 2–5 store 4–6 (class 2) for Ex 5

know that constant composition (f_{ic}), then Eqs. 16a and 26a become linear. Solving such an MILP would yield a solution with no composition discrepancy. Thus, for *each* tank, we can divide all periods into two distinct blocks: one for which we know the tank compositions, and the other for which we do not. For the former, we can use the exact linear constraints ($FCTU_{iuct} = f_{ic}FTU_{iut}$) and for the latter, we just drop Eq. 16a and 26a from our formulation. Second, observe that a tank's composition changes only when it receives crude from a parcel or another tank; otherwise not. During most periods, the tank will receive nothing; thus, its composition will be constant. However, we must know the constant composition to make the nonlinear flow constraint linear. To this end, our knowledge of the initial compositions of tanks comes in handy. Because we know the initial composition in each tank, we can identify one initial block of periods for which the composition is constant and known. The length of this block will vary from tank to tank and it could be as short as just one period for some tanks. However, this at least provides a start for our algorithm. As a first try, we can use the exact linear constraints for these first blocks of periods and linear approximations for the remaining periods and solve the MILP. This would give us a solution that has no composition discrepancy, at least for the first block of periods on each tank. It will also give us the compositions in all tanks at the end of each period in each block. We now identify the first common block of periods for which we know the compositions in all tanks. We freeze the schedule until the end of that block, and repeat the entire procedure for scheduling the remaining periods. In other words, we now solve another scheduling problem with a reduced horizon. In this manner, we obtaint progressively longer and longer partial schedules, free of composition discrepancies, by solving a series of MILPs, until we have the complete schedule. We now describe the algorithm in full detail.

For the time being, we disallow tank-to-tank transfers in describing the algorithm (Figure 5). At each iteration of our algorithm, we divide the NT periods into two sets. Set 1 includes all periods with $t \le t^*$ for some t^*, and set 2 the rest. The schedule (or all variables Y_{iut}, X_{pit}, FTU_{iut}, $FCTU_{iuct}$, etc. in the MILP) for periods in set 1 is (are) fixed based on

Table 9. Model Performance and Statistics for Illustrated Examples

Example	No. of Equations	Single Variables	Discrete Variables	Profit (k$)	CPU Time (s)	Number of Iterations	Periods NT	Relative MILP Gaps % (Periods)
1								
Li et al. (2002)	3247	1285	379	95,904,657	310	3 (NLP + MILP)	7	10%
Proposed	1996	1022	171	105,760,000	94	7 (MILP)	7	10% (1–7)
Proposed	1996	1022	171	105,780,000	327	7 (MILP)	7	0.01% (1–7)
2	4165	2972	304	3425	1364	7 (MILP)	20	5% (1–5), 3.5% (6–10) 2% (11–15), 0% (16–20)
3	8420	6102	589	4593	11,963	10 (MILP)	42	7% (1–17), 4% (18–27) 2% (28–30), 0% (29–42)
4	4705	2383	335	2467	2615	10 (MILP)	15	1% (1–5), 0.5% (6–15)
5	4078	1960	239	2524	1068	5 (MILP)	15	3% (1–3), 2% (4–5), 0.01% (6–15)
6								
No tank transfers	3033	1503	162	1720	12,835	6 (MILP)	10	0% (1–10)
With tank transfers	3579	1831	186	1767.2	41,604	6 (MILP)	10	0% (1–10)

Table 10. Operation Schedule for Example 2 of Li et al. (2002) Obtained by Our Approach*

Tank	Crude Amount [to CDU No.] (from Vessel No) in Tons for Period						
	1	2	3	4	5	6	7
1		+3000(3,4)		+3000(9)		+3000(14,15)	
2	−1000[2]	−3000[2]	−1000[2]		+5000(10,11)	−5000[1]	−5000[1]
					+3500(13,14)		
3	−3000[1]	−4000[1]					
4			+5000(5,6)		−5000[1]	−3000[1]	−3000[1]
			+3000(7,8)				
5	+3000(1,2)		−2000[1]	−3000[2]	−3000[2]	−3000[2]	−3000[2]
6	−5000[1]	−4000[1]	−5000[1]	−3000[1]	−3000[1]		+3000(16,17)
7	−2000[2]		−3000[1]	−5000[1]			

*"−" sign represents delivery to [CDU]; "+" sign represents receipt from (Vessel).

previous iterations. However, the MILP variables for periods in set 2 are free. From the schedule for set 1, we know the tank compositions at the end of period t^* or $t = t^*$. Let f_{ic} denote the fraction of crude c in tank i at $t = t^*$. Then, our iterative algorithm proceeds as follows:

(1) Set $t^* = 0$.

(2) From the fixed schedule for $t \le t^*$, compute f_{ic} for each tank i as $f_{ic} = VCT_{ict^*}/V_{it^*}$ using the known information (VCT_{ict^*} and V_{it^*}).

(3) For each tank i, identify the latest period $t_i^* > t^*$ such that its composition is f_{ic} for all periods $t^* < t \le t_i^*$. We do this as follows. Let p' denote the last parcel that was unloaded (to any tank) before t^*. Let $\Delta PS_{p'}$ denote the amount of crude remaining in parcel p'. If p' has unloaded fully before t^*, then $\Delta PS_{p'} = 0$. Now, let p'' denote the earliest ($p'' \ge p'$) parcel that tank i can possibly receive after t^*. If there is no such parcel, then $t_i^* = NT$. If both p' and p'' belong to the same vessel (VLCC or ship), then we get

$$t_i^* = t^* + \operatorname{ceil}\left(\frac{\Delta PS_{p'}}{\max\limits_{(i,p')\in \boldsymbol{PI}} FPT^U_{p'i}} + \sum_{p=p'+1}^{p''-1} \frac{PS_p}{\max\limits_{(i,p)\in \boldsymbol{PI}} FPT^U_{pi}}\right) \quad (33)$$

where the second term is the minimum time needed to transfer the remainder of parcel p' and all parcels between p' and p''. If p'' belongs to a different vessel, then $t_i^* = ETU_{p''}$.

(4) Add the constraint, $FCTU_{iuct} = f_{ic}FTU_{iut}$, for $t^* < t \le t_i^*$ in the MILP and solve.

(5) Fix the MILP variables for periods $t^* < t \le \min_i[t_i^*]$. Set $t^* = \min_i[t_i^*]$. If $t^* = NT$, then terminate; otherwise go to Step 2.

Note that the size and complexity of MILP in our algorithm reduce progressively by at least one period at each iteration. Although the algorithm does not guarantee an optimal solution, considering the fact that even solving the MILP is a challenging problem, our approach is quite attractive because it does not require solving MINLPs or NLPs and gives near-optimal schedules in reasonable time. In fact, it gives better solutions for several literature problems. When the scheduling objective does not involve crude compositions, then our algorithm guarantees a globally optimal objective value right in the first MILP, although subsequent iterations are required to correct the composition discrepancy. For large and complex problems, solving MILP to zero gap can be compute intensive, and thus we may have to use a small relative optimality gap for limiting

Table 11. Operation Schedule for Example 2

Tank	Crude Amount [to CDU No.] (from Vessel No) in kbbl for Period*									
	1 & 11	2 & 12	3 & 13	4 & 14	5 & 15	6 & 16	7 & 17	8 & 18	9 & 19	10 & 20
1										
&	−20[3]	−20[3]	−20[3]	−20[3]	−28.8[3]	−45[3]	−45[3]	−45[3]	−23.1[3]	−23.1[3]
2			−45[2]	−45[2]	−45[2]	−30[2]	−20[2]	−20[2]	−20[2]	
&				+10(5)		−45[1]	−45[1]	−45[1]	−45[1]	−45[1]
				+250(6)						
3			+190(4)							
&			−20[2]	−20[2]	−45[2]	−45[2]	−45[2]	−45[2]	−45[2]	−45[2]
4	−20[1]	−20[1]	−20[1]	−45[1]	−45[1]	−20[1]	−20[1]	20[1]	−45[1]	−45[1]
	−20[2]	−20[2]								−25[2]
&	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]					
	−25[2]	−25[2]								
5	−20[2]	−20[2]								−20[2]
&	−20[2]	−20[2]				+240(8)				
6		+130(2)			−21.9[3]	−21.9[3]	−21.9[3]	−21.9[3]	−21.9[3]	−21.9[3]
		+270(3)								
&										
7	+10(1)		+30(3)		−20[3]	−20[3]	−20[3]	−20[3]	−20[3]	−20[3]
	+120(2)									
&					+250(7)					
8	−20[3]	−20[3]	−20[3]	−20[3]						
&	−21.1[3]	−21.1[3]	−21.1[3]	−21.1[3]					−21.9[3]	−21.9[3]

*"−" sign represents delivery to [CDU]; "+" sign represents receipt from (parcel).

the computation time. However, as iterations proceed in our algorithm, the problem size diminishes, which allows us to use decreasing relative gaps to achieve better schedules. For example, if a problem has $NT = 30$, then we can use a relative gap of 5% for the first few iterations and 0% for the rest, as the MILP becomes easier to solve.

Our algorithm does not follow a decomposition strategy in which complicating variables are fixed at successive solves. Instead, it imitates the "rolling horizon" approach, commonly used in scheduling and planning problems. We simply freeze a part of the solution after each solve, and solve smaller problems in successive solves. For each solve, we identify a portion of the scheduling horizon, for which the linear relaxation of the MINLP is *exact*. Then, after that solve, we fix the solution for this exact part. The exactness of this part ensures that the solution for that part has no concentration discrepancy. Fixing the solution for the current exact part gives us a new part of the horizon for which the linear relaxation will be exact. Proceeding this way, we maintain feasibility for each part and ensure that we get an overall feasible solution. Thus, we do not think that "hard" infeasibilities that could arise in a decomposition-based heuristic strategy would arise in our solution process.

Tank-to-tank transfers are extremely difficult to handle because they affect the formulation size and algorithm efficiency immensely. First, we need several additional binary variables and constraints in the formulation to allow tank-to-tank transfers. Because any crude in a given class can be in any tank, the crude transfer and balance constraints must be written for all crudes of the given class. Furthermore, a tank-to-tank transfer can occur in any period; therefore, a constant composition block cannot be longer than one period. In other words, the algorithm must progress only one period in each iteration, until the allowable number of transfers is exhausted. It is also not wise to restrict the number of transfers because it may be difficult to decide whether to consume the available transfers early or save them for the future. All these make the problem extremely difficult.

We now demonstrate the efficacy of our solution algorithm using the motivating example and illustrate how our algorithm corrects the discrepancy between the compositions of sent and delivered crudes.

Illustration

In the motivating example discussed earlier, the VLCC arrives at the start of the scheduling horizon. The first parcel ($p = 1$) is the SBM parcel with crude C2 of Class1. The second is with crude C1 of Class1, whereas the third and the fourth carry Class2 crudes. Based on the parcel sizes and maximum possible transfer rates, we set the earliest possible unloading periods for parcels as $ETU_1 = 1$, $ETU_2 = 1$, $ETU_3 = 2$, and $ETU_4 = 2$. For the first iteration, $t^* = 0$, f_{ic} values are as in Table 6. Because no parcel has begun unloading, p' is undefined. Parcel 1 is the earliest parcel from which T1 and T4 can receive crude, so $p'' = 1$ for both and $t_1^* = t_4^* = ETU_1 = 1$. Similarly, parcel 3 is the earliest parcel from which T2 and T3 can receive crude, so $t_2^* = t_3^* = ETU_3 = 2$. Now, we impose $FCTU_{1uc1} = f_{1c}FTU_{1u1}$, $FCTU_{4uc1} = f_{4c}FTU_{4u1}$, $FCTU_{2uc1} = f_{2c}FTU_{2u1}$, $FCTU_{2uc2} = f_{2c}FTU_{2u2}$, $FCTU_{3uc1} = f_{3c}FTU_{3u1}$, and $FCTU_{3uc2} = f_{3c}FTU_{3u2}$ and solve the MILP. The MILP solution gives a profit of \$3879K, which gives us an upper bound on the globally maximum profit for the exact nonlinear problem.

In the second iteration, $t^* = \min_i[t_i^*] = 1$, so we freeze the schedule for the first period and compute f_{ic} as in Table 6. At $t^* = 1$, we find that both parcels 1 and 2 have unloaded fully, therefore $p' = 2$ for T1 and T4, $\Delta PS_{p'} = 0$, and p'' does not exist. So we obtain $t_1^* = t_4^* = 9$. For T2 and T3, $p'' = 3$ and using Eq. 33, we obtain $t_2^* = t_3^* = 2$. Then, we compute f_{ic} for all tanks at $t^* = 1$ and impose the linear composition constraints for period 2 for T2 and T3 and for periods 2–9 for T1 and T4. The MILP solution gives a profit of \$3879K. Continuing the procedure, we get the detailed results in Table 6. Table 6 also shows the reduction in problem size with iterations. The algorithm terminates after four iterations with a final profit of \$3784.33K, which is within 2.44% of the upper bound \$3879K. Because this is a nonconvex MINLP problem, it is difficult to say what the globally best solution is. Considering the fact that the best solution is surely less than \$3879K, and we are achieving a solution within 2.44% of that without using any NLP, we can safely consider our solution as near-optimal.

Before we illustrate our methodology on some real-life problems, a few remarks highlighting the salient features of our formulation are in order.

Remarks

Our proposed formulations and solution approach differ significantly from previous attempts at this problem.

(1) The first major difference is that our formulation allows some features of a continuous-time formulation in that tank-to-tank and parcel-to-tank transfers may start at times other than period endpoints. This obviates partially the need for a continuous-time formulation.

(2) Unlike Li et al. (2002), our solution approach corrects composition discrepancy without solving a single NLP, although the problem is inherently nonlinear. We have already pointed out earlier that their decomposition strategy can fail to obtain a solution. However, our algorithm fixes parcel-to-tank and tank-to-CDU allocations based on corrected compositions, and thus it cannot produce infeasible results.

(3) Although we also model parcel-to-tank connections using bi-index binary variables, our binary variables (XP_{pt} and XT_{it}) are subtly different from those (VT_{vt} and VI_{vi}) of Li et al. (2002). The implications of this, although quite subtle, are important. Whereas a vessel cannot deliver to multiple tanks during the scheduling horizon in Li et al. (2002) formulation, this is possible in our formulation. To prove this point, let us consider the allocation of vessel v to tank i at time t, as given by the variable XW_{vit} in Li et al. (2002). Li et al. (2002) defined VT_{vt} and VI_{vi} as

$$VT_{vt} = \begin{cases} 1 & \text{if vessel } v \text{ is connected at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$VI_{vi} = \begin{cases} 1 & \text{if vessel } v \text{ is connected to tank } i \\ 0 & \text{otherwise} \end{cases}$$

and fixed XW_{vit} using the constraints: $XW_{vit} \geq VT_{vt} + VI_{vi} - 1$, $XW_{vit} \leq VT_{vt}$, and $XW_{vit} \leq VI_{vi}$. Further, they allowed a vessel v to connect to only one tank i at any time t by imposing Σ_i

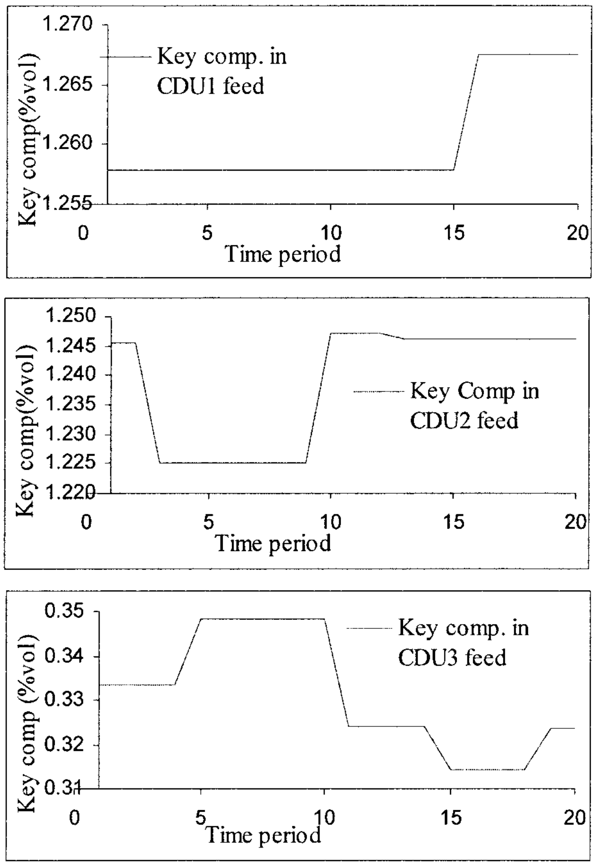

Figure 6. Key component concentration in CDU feeds in various time periods for example 2.

$XW_{vit} \leq 1$. Now, consider a perfectly possible scenario in which a vessel v delivers to tank T1 in period 1 and T2 in period 2. In this case, $VT_{v1} = VT_{v2} = VI_{v1} = VI_{v2} = 1$. The three constraints for fixing XW_{vit} make $XW_{v11} = XW_{v21} = XW_{v12} = XW_{v22} = 1$ and $\Sigma_i XW_{vit} = 2$ for the two periods in violation of the constraint $\Sigma_i XW_{vit} \leq 1$.

(4) A similar comment also holds for tank-to-CDU allocation. Li et al. (2002) used bi-index binary variables (IT_{it} and IL_{il}) to define CD_{ilt} by using $CD_{ilt} \geq IL_{il} + IT_{it} - 1$, $CD_{ilt} \leq IL_{il}$, $CD_{ilt} \leq IT_{it}$, and

$$IT_{it} = \begin{cases} 1 & \text{if tank } i \text{ is connected at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$IL_{il} = \begin{cases} 1 & \text{if tank } i \text{ is connected to CDU } l \\ 0 & \text{otherwise} \end{cases}$$

Suppose that tank T1 delivers to CDU1 in period 1 ($CD_{111} = 1$, $CD_{121} = 0$) and to CDU2 ($CD_{112} = 0$, $CD_{122} = 1$) in period 2. Because T1 must connect to CDU1 and CDU2 both, $IL_{11} = IL_{12} = 1$. Similarly T1 must connect in both periods, so $IT_{11} = IT_{12} = 1$. With these, the defining constraints for CD_{ilt} give $CD_{121} = CD_{112} = 1$, which contradicts the assumed values. In other words, once a tank connects to a set of CDUs, it cannot connect to any other set of CDUs during the scheduling horizon. Even if we were to use bi-index binary variables of the form IT_{it} and LT_{lt} with time dimension in both, it still would not allow all possible charging scenarios. Thus, disaggregating tank-to-CDU binary variables limits the tank-to-CDU connections and does not seem to offer any advantage. Our formulation, on the contrary, allows all viable scenarios.

(5) The existing literature (Lee et al., 1996; Li et al., 2002) defines changeover to arise from a change in composition of feed to CDU. However, it ignores the fact that composition may change, even when flows from two tanks feeding one CDU change. Our Eqs. 22a–22c and 23 accurately describe the transitions. When two tanks deliver to one CDU, then change in flow from any tank also causes a transition. To avoid such a transition, refiners keep the flows from both tanks constant. Our Eqs. 24a and 24b enforce this industry practice.

(6) Unlike our formulation, the changeover constraints of Li et al. (2002) count two changeovers, when a tank stops feeding a CDU and another starts feeding the same. To illustrate this, consider that tank T1 stops feeding and T2 starts feeding to CDU1 from period 3. Thus, the tank-to-CDU allocations are $CD_{112} = CD_{213} = 1$ and $CD_{212} = CD_{113} = 0$. Using the constraints proposed by Li et al. (2002), $Z_{ilt} \geq CD_{ilt} - CD_{il(t-1)}$ and $Z_{ilt} \geq CD_{il(t-1)} - CD_{ilt}$, we find that $Z_{113} = Z_{213} = 1$. In other words, even though only one changeover occurs on CDU1, Li et al. (2002) count them as two, one for each tank.

We now illustrate our methodology on several examples derived from a local refinery.

Examples

We use Example 2 of Li et al. (2002) as our first example (for data, see Li et al., 2002). For the remaining examples (Table 7), we take a refinery with 8 tanks (T1–T8), 3 CDUs (CDU1–CDU3), and two classes (Class1 and Class2) of crudes. Tanks T1, T6, T7, T8, and CDU3 store/process Class1 crudes, whereas the rest store/process Class2 crudes. For Examples 2 to 6, Table 8 gives the initial crude levels in tanks, the crude types, their margins and key component details, the initial crude compositions in tanks, economic data, and limits on crude transfer amounts, and Table 7 gives the tanker arrival details, crude demands, and key component concentration limits on CDUs. We selected these examples to illustrate the use of our models for refineries with different configurations (SBM, jetties, tank-to-tank transfers), short and long scheduling horizons, and several parcel sizes and arrivals. Wherever possible, we compared our approach with the previous approaches for these examples. We used CPLEX 7.0 solver within GAMS on a Gateway E5250 (Pentium II) machine running Windows NT. Table 9 gives the model statistics and histories of model performance for all examples.

Table 12. Solution Details for Example 2

MILP Iteration	Profit (k$)	CPU Time (s)	Relative Gap (%)	
			Target	Actual
1	3427.60	146.5	5.0	4.6
2	3454.86	292.9	3.5	3.4
3	3495.37	836.3	3.5	2.0
4	3444.32	70	3.5	2.8
5	3433.12	7.8	2.0	1.6
6	3439.00	9.8	0.0	0.0
7	3424.90	0.74	0.0	0.0

Table 13. Operation Schedule for Example 4

Tank	Crude Amount [to CDU No.] (from Vessel No) in kbl for Period*														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	−20[3]	−20[3]	−20[3]	−45[3]	−20[3]	−20[3]	−20[3]	−20[3]	−20[3]	−20[3]	−20[3]	−20[3]	−20[3]		
2	−20[2]	−20[2]	−20[2]	−20[2]	−20[2]	−20[2]	−20[2]	−45[2]	−45[2]	+100(6)	+300(8)				
3						+115(3)	+105(5)			−45[2]	−45[2]	−45[2]	−45[2]	−45[2]	−45[2]
4	−20[1]	−20[1]	−20[1]	−20[1]	−20[1]	−20[1]	−20[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]
5						+235(3)	+245(3)		+150(6)						
6			+350(1)			+300(4)									
7				+350(2)							+200(7)				
8					−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−45[3]	−45[3]

*"−" sign represents delivery to [CDU]; "+" sign represents receipt from (parcel).

Example 1

For the sake of a fair comparison, we used the same definition of changeover as used by Li et al. (2002) and a relative gap of 10% for MILPs. Our approach yielded a different schedule (Table 10) with a 10.3% increase in profit (Table 9), compared to that reported by Li et al. (2002). Our approach solved seven MILPs and took 94 s (Table 9), whereas their approach required four (NLP + MILP)s and took 212 s with a profit of 97,902,225 Yuan. We also successfully solved this example using a relative gap of 0.01% without much change in profit in 327 s of CPU time.

Example 2

This example involves one SBM line, twenty 8-h periods, and two VLCCs with three parcels each. Table 11 shows the operation schedule. Some salient features of the schedule are as follows:

(1) T4 with a key component concentration of 0.0126 vol. % cannot meet the feed quality for CDU2, so the optimizer combined crudes from T4 and T5 (0.0123 vol. %) in periods 1, 2, 10, 11, and 12.

(2) Two tanks feed a CDU in several periods. For instance, T6 and T7 feed CDU3 in periods 5–10, T1 and T8 feed CDU3 in periods 11–14, and T4 and T5 feed CDU2 in periods 1, 2 10, and 12. At all times, the optimizer maintained constant individual tank feed flows to avoid a composition change. As mentioned earlier, many refineries practice this.

(3) Sequential, multiple transfers to one or more tanks occur in several periods. For example, in period 1, parcel 1 (SBM) and parcel 2 (1st parcel of VLCC-1) unload to T7. In period 2, parcel 2 and parcel 3 (2nd parcel of VLCC-1) unload to T6. Finally, in period 3, parcel 3 and parcel 4 unload to T7 and T3, respectively. This is a continuous-time feature in our formulation.

(4) T4 delivers to both CDU1 and CDU2 in periods 1, 2, 10, 11, and 12.

(5) T2 receives crude in period 14, uses period 15 for brine settling and removal, and starts delivering only in period 16. T7 does the same in periods 4, 5, and 6.

(6) Finally, we see from Figure 6 that the key component concentration in feed changes, only when a changeover occurs. Table 12 details profit, CPU time, and actual/target relative gaps for iterations.

Example 3

To demonstrate our algorithm's ability to solve larger problems, we consider a longer horizon of 42 periods for Example 2, with three VLCCs carrying three parcels each. For lack of space, we do not report a detailed schedule for this example.

Example 4

In this example, we consider three jetties, no SBM, fifteen 8-h periods, and eight single-parcel vessels (V1–V8). Table 13 shows the operations schedule, whereas Table 14 shows the berth allocations for the arriving vessels. Among some salient features of this schedule, we have two jetties allowing two vessels (V3 and V4) to unload simultaneously to (T3 and T6) and then to (T5 and T6) in period 6. Also in period 6, V3 unloads to both T3 and T5. In period 11, two vessels (V7 and V8) unload simultaneously to two different tanks (T7 and T2, respectively). From periods 5 to 13, T1 and T8 feed CDU3 and we can see that the composition of feed remains the same during this period. The optimizer achieved this by keeping flows from individual tanks constant. Thus, in addition to showing all the features mentioned in Example 1, the schedule shows the simultaneous berth allocations of multiple vessels and simultaneous transfers to multiple tanks in a period.

Example 5

In this example, we consider two jetties, one SBM, fifteen 8-h periods, one VLCC with three parcels, and three single-parcel tankers. Table 15 shows the schedule. Because the refinery has both SBM and multiple jetties, both should be able to transfer crude simultaneously in any period. This example shows this feature in period 5, when parcel 4 from the VLCC and parcel 6 from V2 unload to T2 and T5. Similarly, during periods 6 and 7, parcels 5 and 7 from V1 and V3 simultaneously unload to T7 and T8. We have a few instances of multiple tanks feeding one CDU: T2 and T4 feed CDU1 during periods 11–15 and T2 and T5 feed CDU2 during the same time.

Table 14. Berth Allocation Details for Example 4

Vessle	Jetty Allocation	
	Start Period	End Period
V1	3	3
V2	3	4
V3	6	6
V4	6	7
V5	6	7
V6	9	10
V7	10	11
V8	10	11

Table 15. Operation Schedule for Example 5

Tank	Crude Amount [to CDU No.] (from Vessel No) in kbbl for Period*										
	1	2	3	4	5	6	7	8	9	10	11 to 15
1	−25[3]	−20[3]	−20[3]		−20[3]	−20[3]	−46[3]				
2			+100(2)		+76.8(4)					−37.1[2]	−18.4[1]
					+38.1(6)						−37.1[2]
3			−20[1]	−20[1]	−20[1]	−50[1]	−20[1]	−20[1]	−50[1]	−50[1]	
4	−50[1]	−50[1]			−20[2]	−50[2]	−32.1[2]	−20[2]	−20[2]		−31.5[1]
5	−39.5[2]	−39.5[2]	−39.5[2]	−39.5[2]	+13.2(4)					−12.8[2]	−12.8[2]
					+86.5(6)						
6				+100(3)							
7				−49[3]		+87(5)		−7.7[3]	−7.7[3]	−7.7[3]	−7.7[3]
						+10(7)					
8			+10(1)			+38(5)		−42.3[3]	−42.3[3]	−42.3[3]	−42.3[3]
						+90(7)					

*"−" sign represents delivery to [CDU]; "+" sign represents receipt from (parcel).

Example 6

In this example, we illustrate the benefits of tank transfer operations. We consider one SBM, ten 8-h periods, and one VLCC with three parcels. Table 16 gives the schedules for the case with tank-to-tank transfers and for the case without them. For the former, we allowed at most one tank transfer in periods 1–2 only. Note that the last parcel unloads in period 6 for the latter, whereas the same unloads in period 5 for the former. The optimal schedule for the former shows a transfer of 70 kbbl from T6 to T7 during period 1. This transfer creates the required space and facilitates early unloading of parcel. Thus, the last parcel unloads in period 5 and demurrage is avoided. Furthermore, the profit with transfers is 2.7% greater than that without transfers. Therefore, tank transfers provide additional flexibility to improve profitability.

Conclusions

A discrete-time MINLP model that allows some tasks to begin even at intermediate points in a period and a novel MILP-based solution algorithm that shows no composition discrepancy were developed for the scheduling of crude oil operations in a refinery. In addition to including several real features such as multiple tanks feeding one CDU, one tank feeding multiple CDUs, SBM pipeline, brine settling, tank-to-tank transfers, and so forth, the proposed model uses fewer binary variables and is different from and superior (both in terms of efficiency and quality of solutions) to those reported in previous work. The main feature of our algorithm is that it solves the oil quality, transfer quantity, tank allocation, and oil blending issues simultaneously without solving a single NLP or MINLP. The proposed approach helps quicker and near-optimal decision making in refinery operations and handles problems with up to 14 days. This is a difficult, nonlinear problem and it needs further work to be able to solve problems with longer scheduling horizons to 0% optimality.

Acknowledgments

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Table 16. Comparison of Schedules without and with Tank Transfers for Example 6

Case	Tanks	Crude Amount [to CDU No.] (from Parcel No) {to/from Tank} in kbbl for Period*									
		1	2	3	4	5	6	7	8	9	10
No tank transfer	1			+220(2)		−19.3[3]	−19.3[3]	−19.3[3]	−19.3[3]	−19.3[3]	−19.3[3]
	2										
	3	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]	−14.6[2]
	4	−20[1]	−20[1]	−20[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]	−45[1]
		−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]	−22.9[2]
	5					+40(4)	+400(4)				
	6		+10(1)			−25.7[3]	−25.7[3]	−25.7[3]	−25.7[3]	−25.7[3]	−25.7[3]
			+270(2)								
	7			+10(2)	+400(3)						
	8	−20[3]	−20[3]	−45[3]	−45[3]	+100(3)					
With tank transfer	1	−4.3[3]	−4.3[3]	−4.3[3]	+232.9(3)						
	2				+167.1(4)						
	3	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]	−18.3[1]
	4	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]	−19.1[1]
		−20[2]	−20[2]	−20[2]	−45[2]	−45[2]	−45[2]	−45[2]	−45[2]	−45[2]	−45[2]
	5					+272.9(4)					
	6	−70{7}		+132.9(2)							
				+267.1(3)							
	7	+70{6}	+10(1)		−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−25[3]	−25[3]
			+367.1(2)								
	8	−33.7[3]	−33.7[3]	−33.7[3]	−15.8[3]	−15.8[3]	−15.8[3]	−15.8[3]	−15.8[3]	−15.8[3]	−15.8[3]

*"−" sign represents delivery; "+" sign represents receipt.

Notation

Sets

JP = set of jetty parcels
SP = set of VLCC parcels
PT = set of pairs (parcel p, period t) such that p can connect to SBM line during t
PI = set of pairs (parcel p, tank i) such that i may receive crude from p
IU = set of pairs (tank i, CDU u) such that i can feed crude to CDU u
IC = set of pairs (tank i, crude type c) such that i can hold c
PC = set of pairs (parcel p, crude type c) such that p carries crude c
PV = set of pairs (parcel p, vessel v) such that p is the last parcel of v
II = set of pairs (tank i, tank i') such that transfer between i, i' is allowed

Subscripts

i, i' = storage tanks
c = crude types
u = crude distillation units (CDUs)
v = vessels
t = time periods
p = parcels

Superscripts

U = upper limit
L = lower limit

Parameters

ETA_p = expected time of arrival of parcel p
$FPT_{pi}^{L/U}$ = limits on the amount of crude transfer per period from parcel p to tank i
$FTU_{iu}^{L/U}$ = limits on the amount of crude charge per period from tank i to CDU u
$FTT_{ii'}^{L/U}$ = limits on the amount of crude transfer per period from tank i to i'
$FU_u^{L/U}$ = limits on the amount of crude processed per period by CDU u
$xc_{cu}^{L/U}$ = limits on the composition of crude type c in feed to CDU u
$xk_{ku}^{L/U}$ = limits on the concentration of key component k in feed to CDU u
$V_i^{L/U}$ = allowable limits on crude inventory in tank i
$xt_{ic}^{L/U}$ = limits on the composition of crude type c in tank i
D = total crude demand in the scheduling horizon
D_u = total crude demand per CDU u in the scheduling horizon
D_{ut} = crude demand per CDU u in each period t
PD_j = maximum demand for product j during scheduling horizon
y_{jcu} = fractional yield of product j from crude c in CDU u
CP_{cu} = margin (\$/unit volume) for crude c in CDU u
COC = cost (k\$) per changeover
TTC = penalty (K\$) for occurrence of a tank-to-tank transfer
SSP = safety stock penalty (\$ per unit volume below desired safety stock)
SS = desired safety stock (kbbl) of crude inventory in any period
SWC_v = demurrage or sea waiting cost (\$ per period)
ETD_v = expected time of departure of vessel v
ETU_p = earliest possible unloading period for parcel p
PS_p = size of the parcel p
J = number of identical jetties

Binary variables

XP_{pt} = 1 if parcel p is connected to SBM/jetty discharge line during period t
XT_{it} = 1 if a tank i is connected to SBM/jetty discharge line during period t
Y_{iut} = 1 if a tank i feeds CDU u during period t
$Z_{ii't}$ = 1 if crude transfers between tanks i and i' during period t

0–1 Continuous variables

XF_{pt} = 1 if a parcel p first connects to the SBM/jetty during period t
XL_{pt} = 1 if a parcel p disconnects from the SBM/jetty at time t
X_{pit} = 1 if a parcel p and tank i both connect to the SBM line at t
YY_{iut} = 1 if a tank i is connected to CDU u during both periods t and $(t + 1)$
CO_{ut} = 1 if a CDU u has a changeover during period t

Continuous variables

TF_p = time at which parcel p first connects to SBM/jetty for unloading
TL_p = time at which parcel p disconnects from SBM/jetty after unloading
FPT_{pit} = amount of crude transferred from parcel p to tank i during period t
FTU_{iut} = amount of crude that tank i feeds to CDU u during period t
FU_{ut} = total amount of crude fed to CDU u during period t
$FCTU_{iuct}$ = the amount of crude c delivered by tank i to CDU u during period t
VCT_{ict} = amount of crude c in tank i at the end of period t
V_{it} = crude level in tank i at end of period t
f_{ict} = concentration (volume fraction) of crude c in tank i at the end of period t
DC_v = demurrage cost for vessel v
SC_t = safety stock penalty for period t
ZT_{it} = variable to denote the number of times tank i exchanges crude with another tank in a given period t
$FCTT_{ii'ct}$ = amount of crude c transferred from tank i to tank i' during period t
$FYY_{ii't}$ = total amount of crude transferred from tank i to tank i' during t
$AFTT_{ii't}$ = absolute amount of crude transferred from tank i to tank i' during t

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